# *Approximate solid angle subtended by a circular plane at any point in the space*

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## 1. Introduction:

When a circular with a finite radius is observed from any arbitrary point in the space it always appears as an elliptical plane except its straight-on-positions (i.e. any point lying on the vertical axis passing through the centre of circular plane from which it appears as perfect circular) . Hence, we would derive a general equation using **Approximation Formula for elliptical plane** which can be used for estimating the approximate value of solid angle subtended by a circular plane at any arbitrary point in the space. (See figure 1 below showing a circular plane by a straight line AOB & its projection as an elliptical plane A'MB in the upper view)

## 2. Solid angle subtended by a circular plane at any arbitrary point in the space

Let there be a circular plane with the centre 'O' & radius 'R' and any arbitrary point say 'P' at a distance 'd' from the centre 'O' such that the **angle between normal through the centre 'O' of the plane & the line OP joining the given point 'P' to the centre 'O' is** $'\boldsymbol{\theta}'$ (As shown in the figure 1 below)

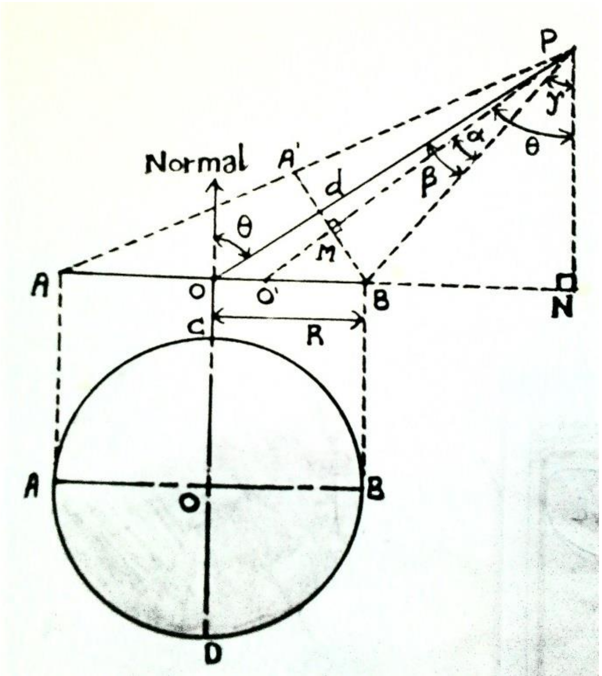


**Figure 1: A circular plane is shown by a straight line AOB with centre O & its projection as an elliptical plane A'MB with centre M in the upper view.** $\boldsymbol{\theta}$ **is the angle between the normal to the plane & the line joining any arbitrary point P, in the space, to the centre O of the circular plane with a radius R.**

$in\ right\ \Delta ONP$

$$\Rightarrow sin\theta = \frac{ON}{OP} = \frac{ON}{d} \Rightarrow ON = dsin\theta \ \& \ cos\theta = \frac{PN}{OP} = \frac{PN}{d} \Rightarrow PN = dcos\theta$$

$$\Rightarrow BN = ON - OB = dsin\theta - R$$

$in\ right\Delta PMB$

$$\Rightarrow PB = \sqrt{PM^2 + MB^2} = \sqrt{h^2 + b^2} = k\ (let)$$

$$where\ \boldsymbol{PM} = \boldsymbol{h}\ \&\ \boldsymbol{MB} = \boldsymbol{A'M} = \boldsymbol{b}\ \ (let's\ assume)$$

$$\Rightarrow \sin\angle BPM = \frac{BM}{BP} \ \Rightarrow sin\alpha = \frac{b}{k} \ \ \&$$

$$cos\angle BPM = \frac{PM}{BP} \qquad \Rightarrow cos\alpha = \frac{h}{k}$$

$in\ right\Delta PNB$

$$\Rightarrow BN^2 + PN^2 = PB^2$$

$$\Rightarrow (dsin\theta - R)^2 + (dcos\theta)^2 = d^2sin^2\theta + R^2 - 2dRsin\theta + d^2cos^2\theta$$

$$\Rightarrow \ \boldsymbol{d^2 + R^2 - 2dRsin\theta = k^2 = h^2 + b^2} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots.(I)$$

$$\Rightarrow sin\angle BPN = \frac{BN}{BP} \Rightarrow sin\gamma = \frac{dsin\theta - R}{k} \ \& \ cos\angle BPM = \frac{PN}{BP} \Rightarrow cos\gamma = \frac{dcos\theta}{k}$$

In $\Delta AOP$ applying **Sine-Rule** as follows

$$\frac{\sin\angle OPA}{AO}=\frac{\sin\angle AOP}{AP}$$

$$\Rightarrow \frac{\sin\left(\alpha-(\theta-(\alpha+\gamma))\right)}{R}=\frac{\sin\left(\frac{\pi}{2}+\theta\right)}{\sqrt{PN^2+AN^2}}=\frac{\cos\theta}{\sqrt{(d\cos\theta)^2+(R+d\sin\theta)^2}}$$

$$\Rightarrow \sin\left((2\alpha+\gamma)-\theta\right)=\frac{R\cos\theta}{\sqrt{d^2\cos^2\theta+R^2+d^2\sin^2\theta+2dR\sin\theta}}=\frac{R\cos\theta}{\sqrt{d^2+R^2+2dR\sin\theta}}$$

$$\sin\left((2\alpha+\gamma)-\theta\right)=l\ (let)\quad where\ \boldsymbol{l}=\frac{\boldsymbol{R\cos\theta}}{\sqrt{\boldsymbol{d^2+R^2+2dR\sin\theta}}}\quad \dots\dots\dots\dots..(II)$$

$$\Rightarrow \sin(2\alpha+\gamma)\cos\theta-\cos(2\alpha+\gamma)\sin\theta=l$$

$$\Rightarrow (\sin2\alpha\cos\gamma+\cos2\alpha\sin\gamma)\cos\theta-(\cos2\alpha\cos\gamma-\sin2\alpha\sin\gamma)\sin\theta=l$$

$$\Rightarrow \left((2\sin\alpha\cos\alpha)\cos\gamma+(2\cos^2\alpha-1)\sin\gamma\right)\cos\theta-\left((2\cos^2\alpha-1)\cos\gamma-(2\sin\alpha\cos\alpha)\sin\gamma\right)\sin\theta=l$$

$$\Rightarrow (2\sin\alpha\cos\alpha\cos\gamma+2\cos^2\alpha\sin\gamma-\sin\gamma)\cos\theta-(2\cos^2\alpha\cos\gamma-\cos\gamma-2\sin\alpha\cos\alpha\sin\gamma)\sin\theta=l$$

*Now on setting the corresponding values, we get*

$$\Rightarrow\left\{2\left(\frac{b}{k}\right)\left(\frac{h}{k}\right)\left(\frac{d\cos\theta}{k}\right)+2\left(\frac{h}{k}\right)^2\left(\frac{d\sin\theta-R}{k}\right)-\left(\frac{d\sin\theta-R}{k}\right)\right\}\cos\theta$$
$$-\left\{2\left(\frac{h}{k}\right)^2\left(\frac{d\cos\theta}{k}\right)-\left(\frac{d\cos\theta}{k}\right)-2\left(\frac{b}{k}\right)\left(\frac{h}{k}\right)\left(\frac{d\sin\theta-R}{k}\right)\right\}\sin\theta=l$$

$$\Rightarrow\quad\left(\frac{2(d\sin\theta-R)\cos\theta}{k^3}-\frac{2d\sin\theta\cos\theta}{k^3}\right)h^2+b\left(\frac{2d\cos^2\theta}{k^3}+\frac{2(d\sin\theta-R)\sin\theta}{k^3}\right)h$$
$$+\left(\frac{d\sin\theta\cos\theta+R\cos\theta-d\sin\theta\cos\theta}{k^3}\right)=l$$

$$\Rightarrow\quad-\left(\frac{2R\cos\theta}{k^3}\right)h^2+2b\left(\frac{d-R\sin\theta}{k^3}\right)h+\left(\frac{R\cos\theta}{k}-l\right)=0$$

**Now, for the convenience, let's assume**

$$\boldsymbol{A}=2\left(\frac{\boldsymbol{R\cos\theta}}{\boldsymbol{k^3}}\right)\qquad \boldsymbol{B}=2\left(\frac{\boldsymbol{d-R\sin\theta}}{\boldsymbol{k^3}}\right)$$
$$\&\ \boldsymbol{C}=\left(\frac{\boldsymbol{R\cos\theta}}{\boldsymbol{k}}-\boldsymbol{l}\right)$$

Thus, we get a biquadratic equation for calculating the value of

$$\Rightarrow -Ah^2+Bbh+C=0\quad\Rightarrow -Ah^2+C=-Bbh\ \ (on\ squaring\ both\ the\ sides)$$

$$\Rightarrow (Ah^2-C)^2=(Bbh)^2\Rightarrow A^2h^4+C^2-2ACh^2=B^2b^2h^2$$

$$\Rightarrow A^2h^4+C^2-2ACh^2=B^2h^2(k^2-h^2)=B^2k^2h^2-B^2h^4\quad (from\ eq(I))$$

$$\Rightarrow\quad \boldsymbol{(A^2+B^2)h^4-(2AC+B^2k^2)h^2+C^2=0}\qquad \dots\dots\dots\dots\dots\dots.(III)$$

$$where, \ \boldsymbol{k} = \sqrt{\boldsymbol{d^2 + R^2 - 2dRsin\theta}} \ \& \ \boldsymbol{l} = \frac{\boldsymbol{Rcos\theta}}{\sqrt{\boldsymbol{d^2 + R^2 + 2dRsin\theta}}} \qquad from\ eq(I)\ \&\ (II)$$

**The above equation is biquadratic in terms of arbitrary variable 'h'. It is called Auxiliary Equation.**

In this case, the **projection of the circular plane,** in the direction of line OP at an angle $'\theta'$ with the normal through the centre 'O' of the circular plane, will be **an elliptical plane** having centre 'M', major axis CD = diameter = 2R & minor axis A'B = 2b (let) when viewed from any arbitrary point P in the space. (As shown in the figure (2) below)

In this case, the solid angle subtended by the given circular plane at the arbitrary point 'P' in the space = solid angle subtended by the elliptical plane of projection of the circular plane at the same point 'P'

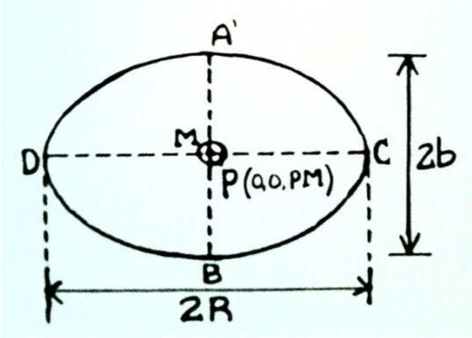

**Figure 2: A circular plane with a radius R, always appearing as an elliptical plane with an imaginary centre except the straight-on-position, is projected as an elliptical plane with centre M, major axis 2R & minor axis 2b.**

Now, by using **HCR's Approximation Formula for the elliptical plane,** we get the approximate value of solid angle subtended by the projected elliptical plane, with major axis 2a & minor axis 2b at a point lying at a distance h on the vertical axis passing through the centre is given as follows

$$\boldsymbol{\omega \cong 2\pi \left[1 - \frac{1}{\sqrt{1 + \left(\frac{ab}{h^2}\right)}}\right]}$$

On setting the values of $a = DM = R, \ \ b = \sqrt{k^2 - h^2} \ \ \& \ \ h = PM \qquad from\ eq(I)$

Hence, we get the **approximate value of solid angle subtended by the circular plane** at the given point in the space, as follows

$$\omega \cong 2\pi \left[1 - \frac{1}{\sqrt{1 + \left(\frac{R\sqrt{k^2 - h^2}}{h^2}\right)}}\right] \qquad (\ on\ setting\ the\ value\ of\ k^2\ from\ eq(I)\ )$$

$$\Rightarrow \boldsymbol{\omega \cong 2\pi \left[1 - \frac{1}{\sqrt{1 + \left(\frac{R\sqrt{d^2 + R^2 - 2dRsin\theta - h^2}}{h^2}\right)}}\right]} \qquad \dots\dots\dots\dots\dots\dots.(IV)$$

**Above is the required expression for calculating the approximate value of solid angle subtended by the circular plane at the given point in the space. The value of $h^2$ is found out from the auxiliary equation (III) given as follows**

$$\boldsymbol{\Rightarrow [(A^2 + B^2)h^4 - (2AC + B^2k^2)h^2 + C^2 = 0] \qquad \forall\ h \geq \sqrt{d^2 - 2dRsin\theta}}$$

Where

$$\boldsymbol{A = 2\left(\frac{R\cos\theta}{(d^2+R^2-2dR\sin\theta)^{\frac{3}{2}}}\right), \quad B = 2\left(\frac{d-R\sin\theta}{(d^2+R^2-2dR\sin\theta)^{\frac{3}{2}}}\right)}$$

$$\boldsymbol{C = R\cos\theta\left(\frac{1}{\sqrt{d^2+R^2-2dR\sin\theta}}-\frac{1}{\sqrt{d^2+R^2+2dR\sin\theta}}\right) \ \& \ k = \sqrt{d^2+R^2-2dR\sin\theta}}$$

**Condition for the appropriate value of arbitrary variable $\boldsymbol{h}$ :** Since, the auxiliary equation is a biquadratic equation in terms of arbitrary variable $h$ or a quadratic equation in terms of $h^2$ hence, it will give two values of $h^2$out of which only one value (appropriate) will be accepted while other value will be discarded by applying the following **condition**

Since, $minor\ axis \le major\ axis \Rightarrow 2b \le 2a \quad or \quad b \le a \Rightarrow \sqrt{k^2-h^2} \le R \quad (since,\ major\ axis, 2a = 2R)$

$$\Rightarrow k^2-h^2 \le R^2 \Rightarrow d^2+R^2-2dR\sin\theta-h^2 \le R^2 \quad (on\ setting\ the\ value\ of\ k)$$

$$h^2 \ge d^2+R^2-2dR\sin\theta-R^2 \quad or \quad \boldsymbol{h \ge \sqrt{d^2-2dR\sin\theta}}$$

**Above is the required condition to decide the appropriate value of arbitrary variable $\boldsymbol{h}$**

**Important deductions:**

If the given point is lying on the normal axis passing through the centre of the circular plane then the solid angle subtended by the circular plane at the same point is obtained by setting $\theta = 0$ in the above eq(IV) as follows

$$\omega \cong 2\pi\left[1-\frac{1}{\sqrt{1+\left(\frac{R\sqrt{d^2+R^2-2dR\sin 0-h^2}}{h^2}\right)}}\right] = 2\pi\left[1-\frac{1}{\sqrt{1+\left(\frac{R\sqrt{d^2+R^2-h^2}}{h^2}\right)}}\right]$$

………………….(V)

The value of $h^2$ is calculated as follows

$$A = 2\left(\frac{R\cos 0}{(d^2+R^2-2dR\sin 0)^{\frac{3}{2}}}\right) = 2\left(\frac{R}{(d^2+R^2)^{\frac{3}{2}}}\right)$$

$$B = 2\left(\frac{d-R\sin 0}{(d^2+R^2-2dR\sin 0)^{\frac{3}{2}}}\right) = 2\left(\frac{d}{(d^2+R^2)^{\frac{3}{2}}}\right) \quad \&$$

$$C = R\cos(0)\left(\frac{1}{\sqrt{d^2+R^2-2dR\sin 0}}-\frac{1}{\sqrt{d^2+R^2+2dR\sin 0}}\right)$$
$$= R\left(\frac{1}{\sqrt{d^2+R^2}}-\frac{1}{\sqrt{d^2+R^2}}\right) = 0\ \&$$

$$k = \sqrt{d^2+R^2-2dR\sin 0} = k = \sqrt{d^2+R^2}$$

Now, on setting the above values in the auxiliary equation, we have

$$(A^2+B^2)h^4+(2AC-B^2k^2)h^2+C^2 = 0$$

$$\Rightarrow \left(4\left(\frac{R}{(d^2+R^2)^{\frac{3}{2}}}\right)^2 + 4\left(\frac{d}{(d^2+R^2)^{\frac{3}{2}}}\right)^2\right)h^4 - \left(4\left(\frac{R}{(d^2+R^2)^{\frac{3}{2}}}\right)(0) + 4\left(\frac{d}{(d^2+R^2)^{\frac{3}{2}}}\right)^2\left(\sqrt{d^2+R^2}\right)^2\right)h^2 + (0)^2 = 0$$

$$\Rightarrow \quad \frac{4(d^2+R^2)}{(d^2+R^2)^3}h^4 - \frac{4d^2(d^2+R^2)}{(d^2+R^2)^3}h^2 = \frac{4}{(d^2+R^2)^2}(h^4 - d^2h^2) = 0$$

$$\Rightarrow \quad (h^2-d^2)h^2 = 0 \quad \Rightarrow \quad \boldsymbol{h^2 = d^2 \;\; \& \;\; h^2 = 0}$$

Thus, **there arise two cases depending on the values of arbitrary variable h** as follows

### Case 1: When the given point is lying on the normal axis through the centre of circular plane at a positive distance from its centre (i.e. $d > 0 \Rightarrow h^2 = d^2$)

Now, on setting $h^2 = d^2$ in the above expression (V), we get

The **approximate solid angle, subtended by the circular plane at any point lying on the normal axis passing through its centre**, is given as follows

$$\Rightarrow \omega \cong 2\pi\left[1 - \frac{1}{\sqrt{1 + \left(\frac{R\sqrt{d^2+R^2-d^2}}{d^2}\right)}}\right] = 2\pi\left[1 - \frac{1}{\sqrt{1+\left(\frac{R^2}{h^2}\right)}}\right] = 2\pi\left[1 - \frac{h}{\sqrt{h^2+R^2}}\right]$$

$$\Rightarrow \omega \cong 2\pi\left[1 - \frac{h}{\sqrt{h^2+R^2}}\right] \quad or \quad \boldsymbol{\omega = 2\pi\left[1 - \frac{h}{\sqrt{h^2+R^2}}\right]}$$

**The above result denotes the exact value of the solid angle subtended by a circular plane at any point lying on the vertical axis passing through its centre. Hence the result is correct.**

### Case 2: When the given point is lying on the circular plane completely inside the boundary (i.e. $d = 0 \Rightarrow h^2 = 0$)

Now, on setting $h^2 = 0$ in the above expression (V), we get

The **approximate solid angle, subtended by the circular plane at any point lying on the normal axis passing through its centre**, is given as

$$\Rightarrow \omega \cong 2\pi\left[1 - \frac{1}{\sqrt{1 + \left(\frac{R\sqrt{d^2+R^2-(0)^2}}{(0)^2}\right)}}\right] = 2\pi\left[1 - \frac{1}{\sqrt{1+\infty}}\right] = 2\pi[1-0]$$

$$\omega \cong 2\pi \quad or \quad \boldsymbol{\omega = 2\pi \; sr}$$

**The above result denotes that solid angle subtended by any plane at any point lying on it within the boundary is always $2\pi\ sr$ i.e. any point lying on a plane within in the boundary, sees that plane all around it**

**3. Dimensions of the elliptical plane of projection for a circular plane when viewed from any arbitrary point (off centre position) in the space:** The **projection of the circular plane,** in the direction of line OP at an angle $'\theta'$ with the normal through the centre 'O' of the circular plane, will be **an elliptical plane** having centre 'M', major axis CD = diameter = 2R & minor axis A'B = 2b when viewed from any arbitrary point P in the space. (As shown in the figure 2 above) All the important dimensions of the elliptical plane of projection are calculated as follows

$$\textbf{Major axis},\ 2a = \textbf{diameter of circular plane} = 2R$$
$$\textbf{Minor axis}, 2b = 2\sqrt{k^2 - h^2}$$

$$\textbf{Eccentricity}, e = \sqrt{1 - \frac{b^2}{a^2}} \qquad (0 \le e < 1 \ \ \forall \ \ b \le a)$$

The **value of $h^2$ is calculated from the auxiliary (biquadratic) equation** given as follows

$$\Rightarrow [(A^2+B^2)h^4 - (2AC + B^2k^2)h^2 + C^2 = 0] \qquad \forall \ \ h \ge \sqrt{d^2 - 2dR\sin\theta}$$

Where

$$A = 2\left(\frac{R\cos\theta}{(d^2 + R^2 - 2dR\sin\theta)^{\frac{3}{2}}}\right), \qquad B = 2\left(\frac{d - R\sin\theta}{(d^2 + R^2 - 2dR\sin\theta)^{\frac{3}{2}}}\right)$$

$$C = R\cos\theta\left(\frac{1}{\sqrt{d^2 + R^2 - 2dR\sin\theta}} - \frac{1}{\sqrt{d^2 + R^2 + 2dR\sin\theta}}\right) \ \& \ \ k = \sqrt{d^2 + R^2 - 2dR\sin\theta}$$

***Above expressions are useful to calculate all the parameters such as major axis, minor axis & eccentricity of the elliptical plane of projection of a circular plane when viewed from any arbitrary point in the space.***

## Illustrative Numerical Example

**Example 1: Calculate major axis, minor axis & eccentricity of the elliptical plane of projection of a circular plane, with a radius 25 units, when viewed from a point at a distance 70 units from the centre such that the angle between normal to the centre & the line joining the given point to the centre of circular plane is $30^o$. Also calculate the approximate value of the solid angle subtended by the circular plane at the same point in the space.**

**Sol.** Given that

$$R = radius\ of\ the\ circular\ plane = 25\ units,$$
$$d = distance\ from\ the\ centre = 70\ units\ \&\ \ \theta = angle\ with\ the\ normal = 30^o$$

First of all let's calculate all the arbitrary constants A, B, C & k with the help of given values as follows

$$\boldsymbol{A} = 2\left(\frac{R\cos\theta}{(d^2 + R^2 - 2dR\sin\theta)^{\frac{3}{2}}}\right) = 2\left(\frac{(25)\cos30^o}{((70)^2 + (25)^2 - 2(70)(25)\sin30^o)^{\frac{3}{2}}}\right) \approx 1.866917824 \times 10^{-4}$$

$$\boldsymbol{B} = 2\left(\frac{d - Rsin\theta}{(d^2 + R^2 - 2dRsin\theta)^{\frac{3}{2}}}\right) = 2\left(\frac{70 - 25sin30^o}{((70)^2 + (25)^2 - 2(70)(25)sin30^o)^{\frac{3}{2}}}\right) \approx 4.958181338 \times 10^{-4}$$

$$\boldsymbol{C} = Rcos\theta\left(\frac{1}{\sqrt{d^2 + R^2 - 2dRsin\theta}} - \frac{1}{\sqrt{d^2 + R^2 + 2dRsin\theta}}\right)$$

$$= 25cos30^o\left(\frac{1}{\sqrt{(70)^2 + (25)^2 - 2(70)(25)sin30^o}} - \frac{1}{\sqrt{(70)^2 + (25)^2 + 2(70)(25)sin30^o}}\right)$$
$$\approx 0.098544198$$

$$\boldsymbol{k} = \sqrt{d^2 + R^2 - 2dRsin\theta} = \sqrt{(70)^2 + (25)^2 - 2(70)(25)sin30^o} \approx 61.44102864$$

The **value of $h^2$ is calculated from the auxiliary (biquadratic) equation** given as follows

$$\Rightarrow (A^2+B^2)h^4 - (2AC + B^2k^2)h^2 + C^2 = 0$$

On substituting the corresponding values, we get

$$((1.866917824 \times 10^{-4})^2 + (4.958181338 \times 10^{-4})^2)h^4 - (2(1.866917824 \times 10^{-4})(0.098544198) + (4.958181338 \times 10^{-4})^2(61.44102864)^2)h^2 + (0.098544198)^2 = 0$$

$$\Rightarrow \boldsymbol{2.806894434 \times 10^{-7}h^4 - 9.648242563 \times 10^{-4}h^2 + 9.710958959 \times 10^{-3} = 0}$$

$$\therefore \boldsymbol{h^2} = \frac{9.648242563 \times 10^{-4} \pm \sqrt{(9.648242563 \times 10^{-4})^2 - 4(2.806894434 \times 10^{-7})(9.710958959 \times 10^{-3})}}{2(2.806894434 \times 10^{-7})}$$

$$= \frac{9.648242563 \times 10^{-4} \pm \sqrt{9.308858455 \times 10^{-7} - 1.090305466 \times 10^{-8}}}{5.613788868 \times 10^{-7}}$$

$$= \frac{9.648242563 \times 10^{-4} \pm 9.591573337 \times 10^{-4}}{5.613788868 \times 10^{-7}}$$

**But, we know that** $h \geq \sqrt{d^2 - 2dRsin\theta}$ $or$ $h \geq \sqrt{(70)^2 - 2(70)(25)sin30^o}$ $or$ $\boldsymbol{h \geq 56.1248608}$

**Case 1: Taking positive sign**

$$\boldsymbol{h^2} = \frac{9.648242563 \times 10^{-4} + 9.591573337 \times 10^{-4}}{5.613788868 \times 10^{-7}} \approx \boldsymbol{3427.242519} \Rightarrow \boldsymbol{h \approx 58.54265555}$$

$\Rightarrow$ $\boldsymbol{h \approx 58.54265555 \geq 56.1248608}$ **hence this value of arbitrary variable $h$ is accepted.**

**Case 2: Taking negative sign**

$$\boldsymbol{h^2} = \frac{9.648242563 \times 10^{-4} - 9.591573337 \times 10^{-4}}{5.613788868 \times 10^{-7}} \approx \boldsymbol{10.09464861} \Rightarrow \boldsymbol{h \approx 3.177207675}$$

**But, $h \geq 56.1248608$ hence this value of arbitrary variable $h$ is discarded.**

Now, all the important dimensions of the elliptical plane of projection are calculated as follows

$$\boldsymbol{Major\ axis},\ \boldsymbol{2a} = diameter\ of\ circular\ plane = 2R = \boldsymbol{2 \times 25 = 50\ units}$$
$$\boldsymbol{Minor\ axis}, \boldsymbol{2b} = 2\sqrt{k^2 - h^2} = 2\sqrt{(61.44102864)^2 - 3427.242519} \approx \boldsymbol{37.29651358\ units}$$

$$\boldsymbol{Eccentricity, e = \sqrt{1 - \frac{(18.64825679)^2}{(25)^2}} \approx 0.666024046 \quad (0 \leq e < 1 \ \forall \ b \leq a)}$$

Hence, the approximate value of the solid angle subtended by the circular plane at the given point in the space is calculated by using approximate formula as follows

$$\boldsymbol{\omega} \cong 2\pi \left[1 - \frac{1}{\sqrt{1 + \left(\frac{R\sqrt{d^2 + R^2 - 2dR\sin\theta} - h^2}{h^2}\right)}}\right]$$

$$\cong 2\pi \left[1 - \frac{1}{\sqrt{1 + \left(\frac{(25)\sqrt{(70)^2 + (25)^2 - 2(70)(25)\sin 30^o} - 3427.242519}{3427.242519}\right)}}\right] \approx \mathbf{0.388168533}\ \boldsymbol{sr}$$

Thus, the given **circular plane with a radius 25 units** subtends solid angle approximately $\mathbf{0.388168533}\ \boldsymbol{sr}$ **at the given point from which it appears as an elliptical plane with major axis $2a = 50$ units, minor axis $2b \approx 37.29651358$ units & an eccentricity $e \approx 0.666024046$**

**Conclusion:** *Above articles are useful for calculating the approximate value of solid angle subtended by a circular plane at any point in the space. These are also applicable for calculating all the parameters such as major axis, minor axis & eccentricity of elliptical plane of projection of a circular plane when viewed from an off-centre position in the space. All the articles have been derived by the author using* ***Approximation Formula*** *which had been derived by the author in his book "****Advanced Geometry****".*

**Note:** Above articles had been derived *& illustrated by* ***Mr H.C. Rajpoot*** ***(B Tech, Mechanical Engineering)***

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