Mathematical Analysis of Tetrahedron

Application of HCR′s Inverse Cosine Formula & Theory of Polygon

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1. Introduction: We very well know that a **tetrahedron is a solid having 4 triangular faces, 6 edges & 4 vertices. Three triangular faces meet together at each of its four vertices & each of its six edges is shared (common) by two triangular faces.** A tetrahedron is specified by the lengths of three edges meeting at any of its four vertices & the values of (three) angles between them consecutively**.** But if all three angles between the consecutive edges meeting at any vertex are known then we can easily calculate the internal (dihedral) angles between the consecutive triangular faces meeting at the same vertex & the solid angle subtended at the same vertex by the given tetrahedron. (See figure 1 showing a tetrahedron PQRS) Here we are to determine the internal (dihedral) angles between the consecutive triangular faces meeting at any of four vertices & the solid angle subtended by the tetrahedron at the same vertex given three angles α, β & γ between the consecutive (lateral) edges meeting at the same vertex.

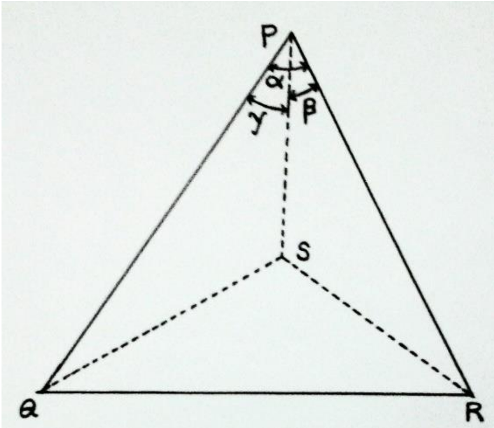

Figure 1: A tetrahedron PQRS having angles $\boldsymbol{\alpha}, \boldsymbol{\beta}$ & $\boldsymbol{\gamma}$ between the consecutive (lateral) edges PQ, PR & PS meeting at the vertex P.

2. Analysis of tetrahedron given the angles $\boldsymbol{\alpha}, \boldsymbol{\beta}$ & $\boldsymbol{\gamma}$ between the consecutive lateral edges: Consider any tetrahedron PQRS having angles α, β & γ between the consecutive lateral edges PQ, PR, & PS ($\forall\ \alpha \le \beta \le \gamma$) meeting at the vertex (apex) P.

Internal (dihedral) angles $\boldsymbol{\theta_1}$, $\boldsymbol{\theta_2}$ & $\boldsymbol{\theta_3}$ between the consecutive lateral triangular faces $\boldsymbol{\Delta PSQ}$ & $\boldsymbol{\Delta PSR}$, $\boldsymbol{\Delta PQR}$ & $\boldsymbol{\Delta PQS}$ and $\boldsymbol{\Delta PRQ}$ & $\boldsymbol{\Delta PRS}$ respectively: We know that the interior angle of between two consecutive triangular faces is measured normal to their common edge. (See figure 1 above)

Now the interior angles θ_1, θ_2 & θ_3 between consecutive (lateral) triangular faces of the tetrahedron PQRS meeting at the vertex P, are determined/calculated by using **HCR's Inverse Cosine Formula** according to which **if x, y & z are the angles between consecutive (lateral) edges meeting at any of four vertices of a tetrahedron then the angle (opposite to α) between two consecutive lateral faces** is given as follows

$$\theta = cos^{-1}\left(\frac{cosx - cosycosz}{sinysinz}\right)$$

Now, setting the corresponding values in the above equation, we get all three interior angles as follows

$$\boldsymbol{\theta_1 = cos^{-1}\left(\frac{cos\alpha - cos\beta cos\gamma}{sin\beta sin\gamma}\right)}$$

$$\boldsymbol{\theta_2 = cos^{-1}\left(\frac{cos\beta - cos\gamma cos\alpha}{sin\gamma sin\alpha}\right)}$$

$$\boldsymbol{\theta_3 = cos^{-1}\left(\frac{cos\gamma - cos\alpha cos\beta}{sin\alpha sin\beta}\right)}$$

Solid angle subtended by the tetrahedron PQRS at the common vertex P: For ease of calculation of the solid angle subtended by tetrahedron PQRS at the common vertex P (figure 1 above), let's cut three equal segments $PA = PB = PC = d$ from the lateral edges PS. PQ & PR respectively. Now, join the points A, B & C by the straight lines to obtain $\Delta \boldsymbol{ABC}$ which exerts a solid angle equal to that subtended by the original tetrahedron PQRS at its vertex P. Thus we would calculate the solid angle subtended by ΔABC at the (common) vertex P by two methods 1) **Analytic** & 2) **Graphical** as given below.

1. Analytic method for calculation of solid angle:

Sides of $\Delta \boldsymbol{ABC}$: Let the sides of ΔABC be a, b & c opposite to its angles A, B & C respectively.

In isosceles ΔPBC

$$\Rightarrow \sin\frac{\angle BPC}{2} = \frac{\left(\frac{BC}{2}\right)}{PB} \quad \Rightarrow \sin\frac{\alpha}{2} = \frac{\left(\frac{a}{2}\right)}{d} \quad \Rightarrow \quad \boldsymbol{a = 2d\sin\frac{\alpha}{2}}$$

$$similarly, \quad \boldsymbol{b = 2d\sin\frac{\beta}{2}} \ \& \ \ \boldsymbol{c = 2d\sin\frac{\gamma}{2}}$$

Now from **HCR's Axiom-2,** we know that **the perpendicular drawn from any vertex of a tetrahedron always passes through circumscribed centre of the (plane) triangle (in this case ΔABC) obtained by joining the points on the lateral edges equidistant from the same vertex.** (See the figure 2)

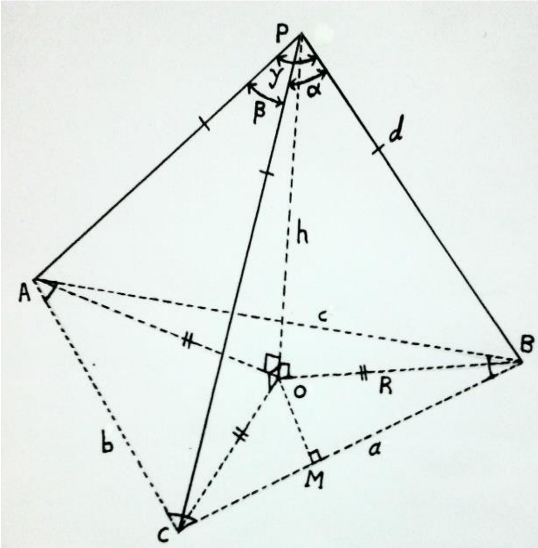

Figure 2: The perpendicular PO drawn from the vertex P of the original tetrahedron PQRS to the plane of ΔABC always passes through circumscribed centre O according to HCR Axiom

Hence, the **circumscribed radius ($\boldsymbol{R}$) of $\Delta \boldsymbol{ABC}$** having its sides a, b & c $(all\ known)$ is calculated as follows

$$s = \frac{a+b+c}{2} = \frac{2d\sin\frac{\alpha}{2} + 2d\sin\frac{\beta}{2} + 2d\sin\frac{\gamma}{2}}{2} = d\left(\sin\frac{\alpha}{2} + \sin\frac{\beta}{2} + \sin\frac{\gamma}{2}\right)$$

$$\Rightarrow Area\ of\ \Delta ABC,\ \Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

Now, by substituting all the corresponding values in the above expression, we get

$$\Delta = \sqrt{d\left(\sin\frac{\alpha}{2} + \sin\frac{\beta}{2} + \sin\frac{\gamma}{2}\right)\left(d\left(\sin\frac{\alpha}{2} + \sin\frac{\beta}{2} + \sin\frac{\gamma}{2}\right) - 2d\sin\frac{\alpha}{2}\right)\left(d\left(\sin\frac{\alpha}{2} + \sin\frac{\beta}{2} + \sin\frac{\gamma}{2}\right) - 2d\sin\frac{\beta}{2}\right)\left(d\left(\sin\frac{\alpha}{2} + \sin\frac{\beta}{2} + \sin\frac{\gamma}{2}\right) - 2d\sin\frac{\gamma}{2}\right)}$$

$$= d^2\sqrt{\left(\sin\frac{\alpha}{2} + \sin\frac{\beta}{2} + \sin\frac{\gamma}{2}\right)\left(\sin\frac{\beta}{2} + \sin\frac{\gamma}{2} - \sin\frac{\alpha}{2}\right)\left(\sin\frac{\alpha}{2} + \sin\frac{\gamma}{2} - \sin\frac{\beta}{2}\right)\left(\sin\frac{\alpha}{2} + \sin\frac{\beta}{2} - \sin\frac{\gamma}{2}\right)}$$

$$= d^2\sqrt{2\left(\sin^2\frac{\alpha}{2}\sin^2\frac{\beta}{2} + \sin^2\frac{\beta}{2}\sin^2\frac{\gamma}{2} + \sin^2\frac{\gamma}{2}\sin^2\frac{\alpha}{2}\right) - \sin^4\frac{\alpha}{2} - \sin^4\frac{\beta}{2} - \sin^4\frac{\gamma}{2}}$$

$$= d^2\sqrt{4\sin^2\frac{\alpha}{2}\sin^2\frac{\beta}{2} - \left(\sin^2\frac{\alpha}{2} + \sin^2\frac{\beta}{2} - \sin^2\frac{\gamma}{2}\right)^2}$$

Hence, the **circumscribed radius (R) of ΔABC** having its sides $a, b \,\&\, c$ $(all\ known)$ is given as follows

$$R = \frac{abc}{4\Delta} = \frac{\left(2d\sin\frac{\alpha}{2}\right)\left(2d\sin\frac{\beta}{2}\right)\left(2d\sin\frac{\gamma}{2}\right)}{4\left(d^2\sqrt{4\sin^2\frac{\alpha}{2}\sin^2\frac{\beta}{2}-\left(\sin^2\frac{\alpha}{2}+\sin^2\frac{\beta}{2}-\sin^2\frac{\gamma}{2}\right)^2}\right)}$$

$$= \frac{2d\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}}{\sqrt{4\sin^2\frac{\alpha}{2}\sin^2\frac{\beta}{2}-\left(\sin^2\frac{\alpha}{2}+\sin^2\frac{\beta}{2}-\sin^2\frac{\gamma}{2}\right)^2}} = Kd\ (let's\ assume)$$

$$\therefore\ \boldsymbol{R = Kd} \quad \boldsymbol{where,\ K = \frac{2\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}}{\sqrt{4\sin^2\frac{\alpha}{2}\sin^2\frac{\beta}{2}-\left(\sin^2\frac{\alpha}{2}+\sin^2\frac{\beta}{2}-\sin^2\frac{\gamma}{2}\right)^2}} = constant} \quad (\boldsymbol{0 < K < 1})$$

$$\forall\ (\boldsymbol{\alpha+\beta}) > \boldsymbol{\gamma},\ (\boldsymbol{\beta+\gamma}) > \boldsymbol{\alpha},\ (\boldsymbol{\gamma+\alpha}) > \boldsymbol{\beta}\ \ \&\ \ (\boldsymbol{\alpha+\beta+\gamma}) < \boldsymbol{360^o}$$

Note: For ease of understanding & calculations always follow the sequence $\alpha \le \beta \le \gamma$ for given (known) values of angles $\alpha,\ \beta \,\&\, \gamma$ between the consecutive lateral edges meeting at a vertex of any tetrahedron.

Hence, the **normal height (h) of ΔABC from the vertex (apex) P of the tetrahedron PQRS** is given as follows

In right ΔPOA

$$PO = \sqrt{(PA)^2-(OA)^2} = \sqrt{d^2-R^2} = \sqrt{d^2-(Kd)^2} = d\sqrt{1-K^2}$$

$$\therefore\ \boldsymbol{h = d\sqrt{1-K^2}}$$

Now, in right $\Delta\boldsymbol{OMB}$

$$\boldsymbol{OM} = \sqrt{(BO)^2-(MB)^2} = \sqrt{R^2-\left(\frac{a}{2}\right)^2} = \sqrt{(Kd)^2-\left(\frac{2d\sin\frac{\alpha}{2}}{2}\right)^2} = d\sqrt{K^2-\sin^2\frac{\alpha}{2}}$$

$$\therefore\ \boldsymbol{OM = d\sqrt{K^2-\sin^2\frac{\alpha}{2}}}$$

Now, from **HCR's Theory of Polygon, the solid angle subtended by the right triangle having its orthogonal sides $\boldsymbol{a}\,\&\,\boldsymbol{b}$ at any point lying at a height $\boldsymbol{h}$ on the vertical axis passing through the vertex common to the side $\boldsymbol{a}$ & the hypotenuse** is given from standard formula as

$$\boldsymbol{\omega = \sin^{-1}\left(\frac{b}{\sqrt{b^2+a^2}}\right) - \sin^{-1}\left\{\left(\frac{b}{\sqrt{b^2+a^2}}\right)\left(\frac{h}{\sqrt{h^2+a^2}}\right)\right\}}$$

Hence, the **solid angle ($\boldsymbol{\omega_{\Delta OBC}}$) subtended by the isosceles $\Delta\boldsymbol{OBC}$ at the vertex P of the tetrahedron**

$$= \omega_{\Delta OMB} + \omega_{\Delta OMC} = 2(\omega_{\Delta OMB}) = 2(solid\ angle\ subtended\ by\ the\ right\ \Delta OMB)$$

$$\Rightarrow\ \omega_{\Delta OBC} = 2\left[\sin^{-1}\left(\frac{(MB)}{\sqrt{(MB)^2+(OM)^2}}\right) - \sin^{-1}\left\{\left(\frac{(MB)}{\sqrt{(MB)^2+(OM)^2}}\right)\left(\frac{(PO)}{\sqrt{(PO)^2+(OM)^2}}\right)\right\}\right]$$

Hence, by setting the corresponding values in the above formula, we get

$$\omega_{\Delta OBC} = 2\left[\sin^{-1}\left(\frac{\left(\frac{a}{2}\right)}{\sqrt{\left(\frac{a}{2}\right)^2 + (OM)^2}}\right) - \sin^{-1}\left\{\left(\frac{\left(\frac{a}{2}\right)}{\sqrt{\left(\frac{a}{2}\right)^2 + (OM)^2}}\right)\left(\frac{(h)}{\sqrt{(h)^2 + (OM)^2}}\right)\right\}\right]$$

$$= 2\left[\sin^{-1}\left(\frac{\left(\frac{2d\sin\frac{\alpha}{2}}{2}\right)}{\sqrt{\left(\frac{2d\sin\frac{\alpha}{2}}{2}\right)^2 + \left(d\sqrt{K^2 - \sin^2\frac{\alpha}{2}}\right)^2}}\right)\right.$$

$$\left. - \sin^{-1}\left\{\left(\frac{\left(\frac{2d\sin\frac{\alpha}{2}}{2}\right)}{\sqrt{\left(\frac{2d\sin\frac{\alpha}{2}}{2}\right)^2 + \left(d\sqrt{K^2 - \sin^2\frac{\alpha}{2}}\right)^2}}\right)\left(\frac{(d\sqrt{1-K^2})}{\sqrt{(d\sqrt{1-K^2})^2 + \left(d\sqrt{K^2 - \sin^2\frac{\alpha}{2}}\right)^2}}\right)\right\}\right]$$

$$= 2\left[\sin^{-1}\left(\frac{d\sin\frac{\alpha}{2}}{\sqrt{d^2\sin^2\frac{\alpha}{2} + K^2d^2 - d^2\sin^2\frac{\alpha}{2}}}\right)\right.$$

$$\left. - \sin^{-1}\left\{\left(\frac{d\sin\frac{\alpha}{2}}{\sqrt{d^2\sin^2\frac{\alpha}{2} + K^2d^2 - d^2\sin^2\frac{\alpha}{2}}}\right)\left(\frac{d\sqrt{1-K^2}}{\sqrt{d^2 - K^2d^2 + K^2d^2 - d^2\sin^2\frac{\alpha}{2}}}\right)\right\}\right]$$

$$= 2\left[\sin^{-1}\left(\frac{d\sin\frac{\alpha}{2}}{Kd}\right) - \sin^{-1}\left\{\left(\frac{d\sin\frac{\alpha}{2}}{Kd}\right)\left(\frac{d\sqrt{1-K^2}}{d\sqrt{1-\sin^2\frac{\alpha}{2}}}\right)\right\}\right]$$

$$= 2\left[\sin^{-1}\left(\frac{\sin\frac{\alpha}{2}}{K}\right) - \sin^{-1}\left\{\left(\frac{\sin\frac{\alpha}{2}}{K}\right)\left(\frac{\sqrt{1-K^2}}{\cos\frac{\alpha}{2}}\right)\right\}\right] = 2\left[\sin^{-1}\left(\frac{\sin\frac{\alpha}{2}}{K}\right) - \sin^{-1}\left(\tan\frac{\alpha}{2}\sqrt{\left(\frac{1}{K}\right)^2 - 1}\right)\right]$$

$$\boldsymbol{\omega_{\Delta OBC} = 2\left[\sin^{-1}\left(\frac{\sin\frac{\alpha}{2}}{K}\right) - \sin^{-1}\left(\tan\frac{\alpha}{2}\sqrt{\left(\frac{1}{K}\right)^2 - 1}\right)\right] = \omega_1\ (let)}$$

Similarly, we have

$$\boldsymbol{\omega_{\Delta OAC} = 2\left[\sin^{-1}\left(\frac{\sin\frac{\beta}{2}}{K}\right) - \sin^{-1}\left(\tan\frac{\beta}{2}\sqrt{\left(\frac{1}{K}\right)^2 - 1}\right)\right] = \omega_2\ \ (let)}$$

$$\boldsymbol{\omega_{\Delta OAB} = 2\left[\sin^{-1}\left(\frac{\sin\frac{\gamma}{2}}{K}\right) - \sin^{-1}\left(\tan\frac{\gamma}{2}\sqrt{\left(\frac{1}{K}\right)^2 - 1}\right)\right] = \omega_3\ \ (let)}$$

Now, we must check out the nature of ΔABC whether it is an acute, a right or an obtuse triangle. Let's assume the largest angle is $\gamma\ among\ known\ values\ of\ \alpha,\ \beta\ \&\ \gamma$ hence we can determine the largest angle C of ΔABC using **cosine formula** as follows

$$cosC = \frac{a^2 + b^2 - c^2}{2ab} = \frac{\left(2dsin\frac{\alpha}{2}\right)^2 + \left(2dsin\frac{\beta}{2}\right)^2 - \left(2dsin\frac{\gamma}{2}\right)^2}{2\left(2dsin\frac{\alpha}{2}\right)\left(2dsin\frac{\beta}{2}\right)} = \frac{sin^2\frac{\alpha}{2} + sin^2\frac{\beta}{2} - sin^2\frac{\gamma}{2}}{2sin\frac{\alpha}{2}sin\frac{\beta}{2}}$$

$$\boldsymbol{\therefore cosC = \frac{sin^2\frac{\alpha}{2} + sin^2\frac{\beta}{2} - sin^2\frac{\gamma}{2}}{2sin\frac{\alpha}{2}sin\frac{\beta}{2}}\quad or\quad C = \cos^{-1}\left(\frac{sin^2\frac{\alpha}{2} + sin^2\frac{\beta}{2} - sin^2\frac{\gamma}{2}}{2sin\frac{\alpha}{2}sin\frac{\beta}{2}}\right)\qquad \forall\,\gamma \geq \beta \geq \alpha}$$

By substituting the known values of angles $\alpha,\ \beta\ \&\ \gamma$ in the above expression, we can directly calculate the value of the largest angle C to check out the nature of ΔABC. **Thus, there arise two cases to calculate the solid angle subtended by the plane ΔABC at the vertex P of tetrahedron PQRS as follows**

Case 1: ΔABC is an acute or a right triangle ($\forall\ \boldsymbol{\gamma \geq \beta \geq \alpha\ \ \&\ \ C \leq 90^o}$):

In this case, the foot point O of the perpendicular drawn from the vertex P to the plane of acute ΔABC lies within or on (in case of right triangle) the boundary of this triangle (See the figure 2 above) **All the values of solid angles $\boldsymbol{\omega_1}$, $\boldsymbol{\omega_2}$ & $\boldsymbol{\omega_3}$ corresponding to the angles $\boldsymbol{\alpha,\ \beta\ \&\ \gamma}$ respectively of a tetrahedron are taken as positive**. Hence, the solid angle (ω) subtended by the tetrahedron PQRS at the vertex P is equal to the solid angle $(\omega_{\Delta ABC})$ subtended by the acute/right ΔABC at the vertex P of tetrahedron which is given as the **sum of magnitudes of solid angles** as follows

$$\omega = \omega_{\Delta ABC} = \omega_{\Delta OBC} + \omega_{\Delta OAC} + \omega_{\Delta OAB} = \omega_1 + \omega_2 + \omega_3$$

$\boldsymbol{\therefore Solid\ angle\ subtended\ by\ the\ tetrahedron\ at\ the\ vertex = \omega = \omega_1 + \omega_2 + \omega_3}$

Case 2: ΔABC is an obtuse triangle ($\forall\ \boldsymbol{\gamma > \beta \geq \alpha\ \ \&\ \ C > 90^o}$):

In this case, the foot point O of the perpendicular drawn from the vertex P to the plane of obtuse ΔABC lies outside the boundary of this triangle. (See the figure 3). In this case, **solid angles $\boldsymbol{\omega_1}$ & $\boldsymbol{\omega_2}$ corresponding to the angles $\boldsymbol{\alpha}$ & $\boldsymbol{\beta}$ respectively are taken as positive while solid angle $\boldsymbol{\omega_3}$ corresponding to the largest angle $\boldsymbol{\gamma}$ of a tetrahedron is taken as negative**. Hence, the solid angle subtended by the tetrahedron PQRS at the vertex P is equal to the solid angle $(\omega_{\Delta ABC})$ subtended by the obtuse ΔABC at the vertex P of tetrahedron which is given as the **algebraic sum of solid angles** as follows

$$\omega = \omega_{\Delta ABC} = \omega_{\Delta OBC} + \omega_{\Delta OAC} - \omega_{\Delta OAB} = \omega_1 + \omega_2 - \omega_3$$

$\boldsymbol{\therefore Solid\ angle\ subtended\ by\ the\ tetrahedron\ at\ the\ vertex = \omega = \omega_1 + \omega_2 - \omega_3}$

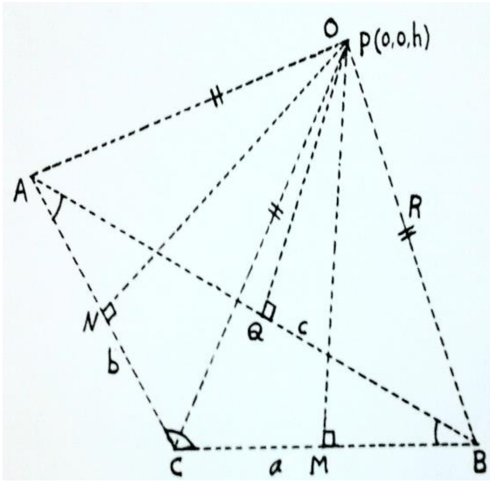

Figure 3: Foot of perpendicular O drawn from the vertex P lies outside the boundary of obtuse ΔABC

2. Graphical method for calculation of solid angle:

In this method, we first plot the diagram of ΔABC having known sides a, b & c by taking a suitable multiplying factor d (as mentioned above) & then specify the location of **foot of perpendicular (F.O.P.)** i.e. the **circumscribed centre** of ΔABC then draw the **perpendiculars from circumscribed centre O to all the opposite sides to divide it (i.e. ΔABC) into elementary right triangles** & use **standard formula-1 of right triangle** for calculating the solid angle subtended by each of the elementary right triangles at the centre of sphere which is given as follows

$$\boldsymbol{\omega = \sin^{-1}\left(\frac{b}{\sqrt{b^2+a^2}}\right) - \sin^{-1}\left\{\left(\frac{b}{\sqrt{b^2+a^2}}\right)\left(\frac{h}{\sqrt{h^2+a^2}}\right)\right\}}$$

Then find out the algebraic sum (ω) of the solid angles subtended by the elementary right triangles at the vertex of the given tetrahedron depending on the nature of the triangle ABC.

$$\therefore \textit{Solid angle subtended by the tetrahedron at the vertex} = \omega = \textit{algebraic sum of } \omega_1,\ \omega_2\ \&\ \omega_3$$

Important deductions: 1. Consider any of eight octants in **3-D co-ordinate system**, if three co-ordinates axes X, Y & Z represent the consecutive edges meeting at the origin (vertex) of a tetrahedron then in this case the planes XY, YZ & ZX will represent the consecutive lateral faces meeting/intersecting at the origin (vertex) & we have

$$\alpha = \beta = \gamma = 90^o\ (\boldsymbol{angle\ between\ any\ two\ orthogonal\ axes\ in\ 3D\ system})$$

Now, all the interior angles θ_1, θ_2 & θ_3 opposite to the angles $\alpha,\ \beta$ & γ respectively between the consecutive lateral faces i.e. **planes XY, YZ & ZX** of the tetrahedron can be easily calculated by using inverse cosine formula as follows

$$\Rightarrow \boldsymbol{\theta_1} = cos^{-1}\left(\frac{cos\alpha - cos\beta cos\gamma}{sin\beta sin\gamma}\right) = cos^{-1}\left(\frac{cos90^o - cos90^o cos90^o}{sin90^o sin90^o}\right) = \boldsymbol{90^o}$$

$$\boldsymbol{\theta_2} = cos^{-1}\left(\frac{cos\beta - cos\gamma cos\alpha}{sin\gamma sin\alpha}\right) = cos^{-1}\left(\frac{cos90^o - cos90^o cos90^o}{sin90^o sin90^o}\right) = \boldsymbol{90^o}$$

$$\boldsymbol{\theta_3} = cos^{-1}\left(\frac{cos\gamma - cos\alpha cos\beta}{sin\alpha sin\beta}\right) = cos^{-1}\left(\frac{cos90^o - cos90^o cos90^o}{sin90^o sin90^o}\right) = \boldsymbol{90^o}$$

The above values show that the angle between any two orthogonal planes in 3-D system is $\mathbf{90^o}$.

Now, calculate the constant K by using the formula as follows

$$K = \frac{2sin\frac{\alpha}{2}sin\frac{\beta}{2}sin\frac{\gamma}{2}}{\sqrt{4sin^2\frac{\alpha}{2}sin^2\frac{\beta}{2} - \left(sin^2\frac{\alpha}{2} + sin^2\frac{\beta}{2} - sin^2\frac{\gamma}{2}\right)^2}}$$

$$\boldsymbol{K} = \frac{2sin\frac{90^o}{2}sin\frac{90^o}{2}sin\frac{90^o}{2}}{\sqrt{4sin^2\frac{90^o}{2}sin^2\frac{90^o}{2} - \left(sin^2\frac{90^o}{2} + sin^2\frac{90^o}{2} - sin^2\frac{90^o}{2}\right)^2}} = \frac{\left(\frac{1}{\sqrt{2}}\right)}{\sqrt{1-\left(\frac{1}{2}\right)^2}} = \frac{\sqrt{2}}{\sqrt{3}} = \sqrt{\frac{\mathbf{2}}{\mathbf{3}}}$$

Now, by substituting all the corresponding values, we get

$$\Rightarrow \boldsymbol{\omega_1 = 2\left[\sin^{-1}\left(\frac{sin\frac{\alpha}{2}}{K}\right) - \sin^{-1}\left(tan\frac{\alpha}{2}\sqrt{\left(\frac{1}{K}\right)^2 - 1}\right)\right]}$$

$$= 2\left[\sin^{-1}\left(\frac{sin\frac{90^o}{2}}{\left(\sqrt{\frac{2}{3}}\right)}\right) - \sin^{-1}\left(tan\frac{90^o}{2}\sqrt{\left(\frac{1}{\left(\sqrt{\frac{2}{3}}\right)}\right)^2 - 1}\right)\right]$$

$$\boldsymbol{\omega_1} = 2\left[\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right] = \boldsymbol{2\left[\frac{\pi}{3} - \frac{\pi}{4}\right] = \frac{\pi}{6}}$$

Similarly, we can find out the following values

$$\Rightarrow \boldsymbol{\omega_2 = \frac{\pi}{6}} \quad \& \ \boldsymbol{\omega_3 = \frac{\pi}{6}}$$

The largest angle of ΔABC is C which is calculated by using cosine formula as follows

$$C = \cos^{-1}\left(\frac{sin^2\frac{\alpha}{2} + sin^2\frac{\beta}{2} - sin^2\frac{\gamma}{2}}{2sin\frac{\alpha}{2}sin\frac{\beta}{2}}\right) = \cos^{-1}\left(\frac{sin^2\frac{90^o}{2} + sin^2\frac{90^o}{2} - sin^2\frac{90^o}{2}}{2sin\frac{90^o}{2}sin\frac{90^o}{2}}\right) = \cos^{-1}\left(\frac{1}{2}\right) = 60^o$$

$$\Rightarrow C = 60^o < 90^o$$

Hence, the **plane ΔABC is an acute angled triangle**.

Hence the **foot of perpendicular (F.O.P.) drawn from the vertex (origin) to the plane of ΔABC will lie within the boundary of ΔABC** (See the figure 2 above) hence, the **solid angle subtended by the tetrahedron at the vertex** is the **sum of the magnitudes of solid angles** as follows

$$\boldsymbol{\omega} = \omega_1 + \omega_2 + \omega_3 = \frac{\pi}{6} + \frac{\pi}{6} + \frac{\pi}{6} = \boldsymbol{\frac{\pi}{2}\ sr} \qquad (\boldsymbol{solid\ angle\ subtended\ by\ each\ octant\ at\ the\ origin})$$

Above value shows that the solid angle subtended by each of eight octants in 3-D co-ordinate system at the origin is $\pi/2\ sr$. It can also be calculated by the following expression

$$\boldsymbol{solid\ angle\ subtended\ by\ each\ octant\ at\ the\ origin = \frac{total\ solid\ angle}{no.of\ octants} = \frac{4\pi}{8} = \frac{\pi}{2}\ sr}$$

2. Consider a regular tetrahedron which has four congruent equilateral triangular faces each three meeting at each of four identical vertices. Hence, for any vertex of a regular tetrahedron, we have

$$\alpha = \beta = \gamma = 60^o\ (\boldsymbol{angle\ between\ any\ two\ consecutive\ edges\ of\ a\ regular\ tetrahedron})$$

Now, all the interior angles $\theta_1, \theta_2\ \&\ \theta_3$ opposite to the angles $\alpha,\ \beta\ \&\ \gamma$ respectively between the consecutive equilateral triangular faces of the regular tetrahedron can be easily calculated by using inverse cosine formula as follows

$$\Rightarrow \boldsymbol{\theta_1} = cos^{-1}\left(\frac{cos\alpha - cos\beta cos\gamma}{sin\beta sin\gamma}\right) = cos^{-1}\left(\frac{cos60^o - cos60^o cos60^o}{sin60^o sin60^o}\right) = \cos^{-1}\left(\frac{1}{3}\right) \approx \boldsymbol{70.52877937^o}$$

$$\boldsymbol{\theta_2} = cos^{-1}\left(\frac{cos\beta - cos\gamma cos\alpha}{sin\gamma sin\alpha}\right) = cos^{-1}\left(\frac{cos60^o - cos60^o cos60^o}{sin60^o sin60^o}\right) = \cos^{-1}\left(\frac{1}{3}\right) \approx \boldsymbol{70.52877937^o}$$

$$\boldsymbol{\theta_3} = cos^{-1}\left(\frac{cos\gamma - cos\alpha cos\beta}{sin\alpha sin\beta}\right) = cos^{-1}\left(\frac{cos60^o - cos60^o cos60^o}{sin60^o sin60^o}\right) = \cos^{-1}\left(\frac{1}{3}\right) \approx \mathbf{70.52877937^o}$$

The above values show that the dihedral angle between any two consecutive equilateral triangular faces of a regular tetrahedron is $\cos^{-1}(1/3) \approx 70.52877937^o$.

Now, calculate the constant K by using the formula as follows

$$K = \frac{2sin\frac{\alpha}{2}sin\frac{\beta}{2}sin\frac{\gamma}{2}}{\sqrt{4sin^2\frac{\alpha}{2}sin^2\frac{\beta}{2} - \left(sin^2\frac{\alpha}{2} + sin^2\frac{\beta}{2} - sin^2\frac{\gamma}{2}\right)^2}}$$

$$\boldsymbol{K} = \frac{2sin\frac{60^o}{2}sin\frac{60^o}{2}sin\frac{60^o}{2}}{\sqrt{4sin^2\frac{60^o}{2}sin^2\frac{60^o}{2} - \left(sin^2\frac{60^o}{2} + sin^2\frac{60^o}{2} - sin^2\frac{60^o}{2}\right)^2}} = \frac{\left(\frac{1}{4}\right)}{\sqrt{\frac{1}{4} - \left(\frac{1}{4}\right)^2}} = \frac{\mathbf{1}}{\sqrt{\mathbf{3}}}$$

Now, by substituting all the corresponding values, we get

$$\Rightarrow \boldsymbol{\omega_1} = \mathbf{2}\left[\sin^{-1}\left(\frac{sin\frac{\alpha}{2}}{K}\right) - \sin^{-1}\left(tan\frac{\alpha}{2}\sqrt{\left(\frac{1}{K}\right)^2 - 1}\right)\right]$$

$$= 2\left[\sin^{-1}\left(\frac{sin\frac{60^o}{2}}{\left(\frac{1}{\sqrt{3}}\right)}\right) - \sin^{-1}\left(tan\frac{60^o}{2}\sqrt{\left(\frac{1}{\left(\frac{1}{\sqrt{3}}\right)}\right)^2 - 1}\right)\right]$$

$$\boldsymbol{\omega_1} = 2\left[\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(\sqrt{\frac{2}{3}}\right)\right] = \mathbf{2}\left[\frac{\boldsymbol{\pi}}{\mathbf{3}} - \sin^{-1}\left(\sqrt{\frac{\mathbf{2}}{\mathbf{3}}}\right)\right]$$

Similarly, we can find out the following values

$$\Rightarrow \boldsymbol{\omega_2} = \mathbf{2}\left[\frac{\boldsymbol{\pi}}{\mathbf{3}} - \sin^{-1}\left(\sqrt{\frac{\mathbf{2}}{\mathbf{3}}}\right)\right] \quad \& \quad \boldsymbol{\omega_3} = \mathbf{2}\left[\frac{\boldsymbol{\pi}}{\mathbf{3}} - \sin^{-1}\left(\sqrt{\frac{\mathbf{2}}{\mathbf{3}}}\right)\right]$$

The largest angle of ΔABC is C which is calculated by using cosine formula as follows

$$C = \cos^{-1}\left(\frac{sin^2\frac{\alpha}{2} + sin^2\frac{\beta}{2} - sin^2\frac{\gamma}{2}}{2sin\frac{\alpha}{2}sin\frac{\beta}{2}}\right) = \cos^{-1}\left(\frac{sin^2\frac{60^o}{2} + sin^2\frac{60^o}{2} - sin^2\frac{60^o}{2}}{2sin\frac{60^o}{2}sin\frac{60^o}{2}}\right) = \cos^{-1}\left(\frac{1}{2}\right) = 60^o$$

$$\Rightarrow C = 60^o < 90^o$$

Hence, the **plane ΔABC is an acute angled triangle**.

Hence the **foot of perpendicular (F.O.P.) drawn from the vertex of tetrahedron to the plane of ΔABC will lie within the boundary of ΔABC** (See the figure 2 above) hence, the **solid angle subtended by the tetrahedron at the vertex** is the **sum of the magnitudes of solid angles** as follows

$$\boldsymbol{\omega} = \omega_1 + \omega_2 + \omega_3 = 2\left[\frac{\pi}{3} - \sin^{-1}\left(\sqrt{\frac{2}{3}}\right)\right] + 2\left[\frac{\pi}{3} - \sin^{-1}\left(\sqrt{\frac{2}{3}}\right)\right] + 2\left[\frac{\pi}{3} - \sin^{-1}\left(\sqrt{\frac{2}{3}}\right)\right]$$

$$= \boldsymbol{2\pi - 6\sin^{-1}\left(\sqrt{\frac{2}{3}}\right) \approx 0.551285598\ sr}$$

Above value shows that the solid angle subtended by a regular tetrahedron at any of its four vertices is $\boldsymbol{0.551285598\ sr}$. It can also be calculated by HCR's standard formula of solid angle as follows

$$\boldsymbol{\omega = 2\pi - 2n\sin^{-1}\left(\cos\frac{\pi}{n}\sqrt{\tan^2\frac{\pi}{n} - \tan^2\frac{\alpha}{2}}\right)}$$

For a regular tetrahedron, we have

$$n = no.\,of\ sides\ in\ each\ face = 3 \quad \& \quad \alpha = angle\ between\ lateral\ edges = 60^o$$

$$\therefore\ \boldsymbol{\omega} = 2\pi - 2(3)\sin^{-1}\left(\cos\frac{\pi}{3}\sqrt{\tan^2\frac{\pi}{3} - \tan^2\frac{60^o}{2}}\right) = 2\pi - 6\sin^{-1}\left(\frac{1}{2}\sqrt{\left(\sqrt{3}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2}\right)$$

$$= 2\pi - 6\sin^{-1}\left(\frac{1}{2}\sqrt{\frac{8}{3}}\right) = \boldsymbol{2\pi - 6\sin^{-1}\left(\sqrt{\frac{2}{3}}\right) \approx 0.551285598\ sr}$$

Both the above results are equal hence the generalized formula of a tetrahedron is verified.

$\boldsymbol{Illustrative\ Numerical\ Examples}$

These examples are based on all above articles which are very practical and directly & simply applicable to calculate the dihedral angles between the consecutive faces & the solid angle subtended by the tetrahedron at the vertex. For **ease of understanding & calculations, value of angle $\boldsymbol{\gamma}$ of $\boldsymbol{\Delta ABC}$ is taken as the largest one**).

Example 1: Calculate the interior angles between the consecutive faces & the solid angle subtended by a tetrahedron at the vertex such that the angles between the consecutive edges meeting at the same vertex are $\boldsymbol{30^o, 40^o}$ & $\boldsymbol{50^o}$.

Sol. Here, we have

$$\alpha = 30^o\ ,\ \ \beta = 40^o\ \ \&\ \gamma = 50^o\ \Rightarrow\ \theta_1, \theta_2\ \&\ \theta_3 = ?\ \&\ solid\ angle,\ \ \omega = ?$$

Now, all the interior angles θ_1, θ_2 & θ_3 opposite to the angles α, β & γ respectively of the tetrahedron can be easily calculated by using inverse cosine formula as follows

$$\Rightarrow\ \boldsymbol{\theta_1} = cos^{-1}\left(\frac{cos\alpha - cos\beta cos\gamma}{sin\beta sin\gamma}\right) = cos^{-1}\left(\frac{cos30^o - cos40^o cos50^o}{sin40^o sin50^o}\right) \approx \boldsymbol{40.64407403^o}$$
$$\approx \boldsymbol{40^o 38' 38.67''}$$

$$\boldsymbol{\theta_2} = cos^{-1}\left(\frac{cos\beta - cos\gamma cos\alpha}{sin\gamma sin\alpha}\right) = cos^{-1}\left(\frac{cos40^o - cos50^o cos30^o}{sin50^o sin30^o}\right) \approx \boldsymbol{56.86341165^o \approx 56^o 51' 48.28''}$$

$$\boldsymbol{\theta_3} = cos^{-1}\left(\frac{cos\gamma - cos\alpha cos\beta}{sin\alpha sin\beta}\right) = cos^{-1}\left(\frac{cos50^o - cos30^o cos40^o}{sin30^o sin40^o}\right) \approx \mathbf{93.6796444^o} \approx \mathbf{93^o 40' 46.72''}$$

Now, calculate the constant K by using the formula as follows

$$K = \frac{2sin\frac{\alpha}{2}sin\frac{\beta}{2}sin\frac{\gamma}{2}}{\sqrt{4sin^2\frac{\alpha}{2}sin^2\frac{\beta}{2} - \left(sin^2\frac{\alpha}{2} + sin^2\frac{\beta}{2} - sin^2\frac{\gamma}{2}\right)^2}}$$

Now, by substituting all the corresponding values, we get

$$K = \frac{2sin\frac{30^o}{2}sin\frac{40^o}{2}sin\frac{50^o}{2}}{\sqrt{4sin^2\frac{30^o}{2}sin^2\frac{40^o}{2} - \left(sin^2\frac{30^o}{2} + sin^2\frac{40^o}{2} - sin^2\frac{50^o}{2}\right)^2}} \approx 0.422811997$$

$$\Rightarrow \boldsymbol{\omega_1 = 2\left[\sin^{-1}\left(\frac{sin\frac{\alpha}{2}}{K}\right) - \sin^{-1}\left(tan\frac{\alpha}{2}\sqrt{\left(\frac{1}{K}\right)^2 - 1}\right)\right]}$$

$$\therefore \boldsymbol{\omega_1} = 2\left[\sin^{-1}\left(\frac{sin\frac{30^o}{2}}{0.422811997}\right) - \sin^{-1}\left(tan\frac{30^o}{2}\sqrt{\left(\frac{1}{0.422811997}\right)^2 - 1}\right)\right] \approx \mathbf{0.094028018\ sr}$$

$$\Rightarrow \boldsymbol{\omega_2 = 2\left[\sin^{-1}\left(\frac{sin\frac{\beta}{2}}{K}\right) - \sin^{-1}\left(tan\frac{\beta}{2}\sqrt{\left(\frac{1}{K}\right)^2 - 1}\right)\right]}$$

$$\therefore \boldsymbol{\omega_2} = 2\left[\sin^{-1}\left(\frac{sin\frac{40^o}{2}}{0.422811997}\right) - \sin^{-1}\left(tan\frac{40^o}{2}\sqrt{\left(\frac{1}{0.422811997}\right)^2 - 1}\right)\right] \approx \mathbf{0.094962792\ sr}$$

$$\Rightarrow \boldsymbol{\omega_3 = 2\left[\sin^{-1}\left(\frac{sin\frac{\gamma}{2}}{K}\right) - \sin^{-1}\left(tan\frac{\gamma}{2}\sqrt{\left(\frac{1}{K}\right)^2 - 1}\right)\right]}$$

$$\therefore \boldsymbol{\omega_3} = 2\left[\sin^{-1}\left(\frac{sin\frac{50^o}{2}}{0.422811997}\right) - \sin^{-1}\left(tan\frac{50^o}{2}\sqrt{\left(\frac{1}{0.422811997}\right)^2 - 1}\right)\right] \approx \mathbf{0.00626143758\ sr}$$

The largest angle of ΔABC is C which is calculated by using cosine formula as follows

$$C = \cos^{-1}\left(\frac{sin^2\frac{\alpha}{2} + sin^2\frac{\beta}{2} - sin^2\frac{\gamma}{2}}{2sin\frac{\alpha}{2}sin\frac{\beta}{2}}\right) = \cos^{-1}\left(\frac{sin^2\frac{30^o}{2} + sin^2\frac{40^o}{2} - sin^2\frac{50^o}{2}}{2sin\frac{30^o}{2}sin\frac{40^o}{2}}\right) \approx 88.26545646^o < 90^o$$

Hence, the **plane ΔABC is an acute angled triangle**.

Hence the **foot of perpendicular (F.O.P.) drawn from the vertex of tetrahedron to the plane of ΔABC will lie within the boundary of ΔABC** (See the figure 2 above) hence, the **solid angle subtended by the tetrahedron at the vertex** is the **sum of the magnitudes of solid angles** as follows

$$\boldsymbol{\omega} = \omega_1 + \omega_2 + \omega_3 \approx 0.094028018 + 0.094962792 + 0.00626143758 \approx \mathbf{0.195252247\ sr} \qquad \boldsymbol{Ans}$$

The above value of area implies that the given **tetrahedron** subtends a **solid angle** $\approx \mathbf{0.195252247}\ \boldsymbol{sr}$ at the vertex irrespective of its geometrical dimensions**.**

Example 2: **Calculate the interior angles between the consecutive faces & the solid angle subtended by a tetrahedron at the vertex such that the angles between the consecutive edges meeting at the same vertex are $\mathbf{40^o}, \mathbf{70^o}$ & $\mathbf{85^o}$.**

Sol. Here, we have

$$\alpha = 40^o\ ,\ \ \beta = 70^o\ \ \&\ \gamma = 85^o\ \Rightarrow\ \theta_1, \theta_2\ \&\ \theta_3 = ?\ \&\ \ solid\ angle,\ \ \omega = ?$$

Now, all the interior angles θ_1, θ_2 & θ_3 opposite to the angles $\alpha,\ \beta$ & γ respectively of the tetrahedron can be easily calculated by using inverse cosine formula as follows

$$\Rightarrow\ \boldsymbol{\theta_1} = cos^{-1}\left(\frac{cos\alpha - cos\beta cos\gamma}{sin\beta sin\gamma}\right) = cos^{-1}\left(\frac{cos40^o - cos70^o cos85^o}{sin70^o sin85^o}\right) \approx \mathbf{38.1424026^o} \approx \mathbf{38^o 8' 32.65''}$$

$$\boldsymbol{\theta_2} = cos^{-1}\left(\frac{cos\beta - cos\gamma cos\alpha}{sin\gamma sin\alpha}\right) = cos^{-1}\left(\frac{cos70^o - cos85^o cos40^o}{sin85^o sin40^o}\right) \approx \mathbf{64.54154954^o} \approx \mathbf{64^o 32' 29.58''}$$

$$\boldsymbol{\theta_3} = cos^{-1}\left(\frac{cos\gamma - cos\alpha cos\beta}{sin\alpha sin\beta}\right) = cos^{-1}\left(\frac{cos85^o - cos40^o cos70^o}{sin40^o sin70^o}\right) \approx \mathbf{106.8262695^o}$$
$$\approx \mathbf{106^o 49' 34.57''}$$

Now, calculate the constant K by using the formula as follows

$$K = \frac{2sin\frac{\alpha}{2}sin\frac{\beta}{2}sin\frac{\gamma}{2}}{\sqrt{4sin^2\frac{\alpha}{2}sin^2\frac{\beta}{2} - \left(sin^2\frac{\alpha}{2} + sin^2\frac{\beta}{2} - sin^2\frac{\gamma}{2}\right)^2}}$$

Now, by substituting all the corresponding values, we get

$$K = \frac{2sin\frac{40^o}{2}sin\frac{70^o}{2}sin\frac{85^o}{2}}{\sqrt{4sin^2\frac{40^o}{2}sin^2\frac{70^o}{2} - \left(sin^2\frac{40^o}{2} + sin^2\frac{70^o}{2} - sin^2\frac{85^o}{2}\right)^2}} \approx 0.675830167$$

$$\Rightarrow\ \boldsymbol{\omega_1} = \mathbf{2}\left[\mathbf{sin^{-1}}\left(\frac{\boldsymbol{sin\frac{\alpha}{2}}}{\boldsymbol{K}}\right) - \mathbf{sin^{-1}}\left(\boldsymbol{tan\frac{\alpha}{2}}\sqrt{\left(\frac{\mathbf{1}}{\boldsymbol{K}}\right)^2 - \mathbf{1}}\right)\right]$$

$$\therefore\ \boldsymbol{\omega_1} = 2\left[\sin^{-1}\left(\frac{sin\frac{40^o}{2}}{0.675830167}\right) - \sin^{-1}\left(tan\frac{40^o}{2}\sqrt{\left(\frac{1}{0.675830167}\right)^2 - 1}\right)\right] \approx \mathbf{0.244883226}\ \boldsymbol{sr}$$

$$\Rightarrow\ \boldsymbol{\omega_2} = \mathbf{2}\left[\mathbf{sin^{-1}}\left(\frac{\boldsymbol{sin\frac{\beta}{2}}}{\boldsymbol{K}}\right) - \mathbf{sin^{-1}}\left(\boldsymbol{tan\frac{\beta}{2}}\sqrt{\left(\frac{\mathbf{1}}{\boldsymbol{K}}\right)^2 - \mathbf{1}}\right)\right]$$

$$\therefore\ \boldsymbol{\omega_2} = 2\left[\sin^{-1}\left(\frac{sin\frac{70^o}{2}}{0.675830167}\right) - \sin^{-1}\left(tan\frac{70^o}{2}\sqrt{\left(\frac{1}{0.675830167}\right)^2 - 1}\right)\right] \approx \mathbf{0.289166683}\ \boldsymbol{sr}$$

$$\Rightarrow \boldsymbol{\omega_3 = 2\left[\sin^{-1}\left(\frac{sin\frac{\gamma}{2}}{K}\right) - \sin^{-1}\left(tan\frac{\gamma}{2}\sqrt{\left(\frac{1}{K}\right)^2 - 1}\right)\right]}$$

$$\therefore \boldsymbol{\omega_3} = 2\left[\sin^{-1}\left(\frac{sin\frac{85^o}{2}}{0.675830167}\right) - \sin^{-1}\left(tan\frac{85^o}{2}\sqrt{\left(\frac{1}{0.675830167}\right)^2 - 1}\right)\right] \approx \boldsymbol{0.018999357\ sr}$$

The largest angle of ΔABC is C which is calculated by using cosine formula as follows

$$C = \cos^{-1}\left(\frac{sin^2\frac{\alpha}{2} + sin^2\frac{\beta}{2} - sin^2\frac{\gamma}{2}}{2sin\frac{\alpha}{2}sin\frac{\beta}{2}}\right) = \cos^{-1}\left(\frac{sin^2\frac{40^o}{2} + sin^2\frac{70^o}{2} - sin^2\frac{85^o}{2}}{2sin\frac{40^o}{2}sin\frac{70^o}{2}}\right) \approx 91.52686653^o > 90^o$$

Hence, the **plane $\Delta \boldsymbol{ABC}$ is an obtuse angled triangle**.

Hence the **foot of perpendicular (F.O.P.) drawn from the vertex of tetrahedron to the plane of $\Delta \boldsymbol{ABC}$ will lie outside the boundary of plane $\Delta \boldsymbol{ABC}$** (See the figure 3 above) hence, the **solid angle subtended by the tetrahedron at the vertex** is the **algebraic sum of the solid angles** as follows

$$\boldsymbol{\omega} = \omega_1 + \omega_2 - \omega_3 \approx 0.244883226 + 0.289166683 - 0.018999357 \approx \boldsymbol{0.515050552\ sr} \qquad \boldsymbol{Ans}$$

The above value of area implies that the given **tetrahedron** subtends a **solid angle** $\approx \boldsymbol{0.515050552\ sr}$ at the vertex irrespective of its geometrical dimensions**.**

Conclusion: *All the articles above have been derived by **Mr H.C. Rajpoot** by using **simple geometry & trigonometry**. All above articles (formula) are very practical & simple to apply in case of **any tetrahedron** to calculate the internal (dihedral) angles between the consecutive lateral faces meeting at any of four vertices & the solid angle subtended by it (tetrahedron) at the vertex when the angles between the consecutive edges meeting at the same vertex are known. These are the generalised formula which can also be applied in case of three faces meeting at the vertex of various regular & uniform polyhedrons to calculate the solid angle subtended by the polyhedron at its vertex.*

Note: *Above articles had been derived & illustrated by **Mr H.C. Rajpoot** (**B Tech, Mechanical Engineering**)*

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