Table of solid angles subtended by Archimedean solids (uniform polyhedrons) at their vertices

Mr Harish Chandra Rajpoot *April, 2015*

M.M.M. University of Technology, Gorakhpur-273010 (UP), India

The table, of the standard values of solid angles subtended by all 13 Archimedean solids at their vertices, has been prepared by the author Mr H.C. Rajpoot using **standard formula of solid angle** & **formula of tetrahedron** for known values of $\boldsymbol{\alpha}, \boldsymbol{\beta}$ & $\boldsymbol{\gamma}$. These values are very useful for the purpose of analysis of Archimedean solids. The author has assumed that the eye of the observer is located at one of the identical vertices of an Archimedean solid & directed (focused) straight to the centre of that solid. In-fact, the solid angle, subtended by an Archimedean solid at its vertex, merely depends on the geometry of the faces & the no. of faces meeting at each vertex. Thus an observer can easily visualise each solid from its vertex but absolute values of solid angle & their relative difference is not possible by simple observation. Hence the mathematical expressions of exact values of solid angle subtended by all 13 Archimedean solids have been tabulated below.

Table of solid angles subtended by all 13 Archimedean solids (uniform polyhedrons) at their vertices

S/N	Archimedean solid	Solid angle subtended by Archimedean solid at each of its identical vertices (in Ste-radian (sr))
1	Truncated tetrahedron $(\alpha = 60^o, \beta = 120^o, \gamma = 120^o)$	$2\sin^{-1}\left(\frac{5(\sqrt{11}-\sqrt{2})}{18\sqrt{3}}\right) + 4\sin^{-1}\left(\frac{\sqrt{11}-\sqrt{2}}{6}\right) = 2\cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \approx 1.910633236\ sr$
2	Truncated hexahedron (cube) $(\alpha = 60^o, \beta = 135^o, \gamma = 135^o)$	$2\sin^{-1}\left(\frac{\sqrt{7+4\sqrt{2}}-1}{4\sqrt{3}}\right) + 4\sin^{-1}\left(\frac{\sqrt{6+\sqrt{2}}-\sqrt{2-\sqrt{2}}}{4}\right) \approx 2.801755744\ sr$
3	Truncated octahedron $(\alpha = 90^o, \beta = 120^o, \gamma = 120^o)$	$2\sin^{-1}\left(\frac{2\sqrt{10}-2}{9}\right) + 4\sin^{-1}\left(\frac{\sqrt{10}-1}{3\sqrt{2}}\right) = \pi \approx 3.141592654\ sr$
4	Truncated dodecahedron $(\alpha = 60^o, \beta = 144^o, \gamma = 144^o)$	$2\sin^{-1}\left(\frac{\sqrt{173-9\sqrt{5}}-\sqrt{58-24\sqrt{5}}}{10\sqrt{6}}\right) + 4\sin^{-1}\left(\frac{\sqrt{20(9+2\sqrt{5})}-5+\sqrt{5}}{20}\right)$ $\approx 3.87132031\ sr$
5	Truncated icosahedron $(\alpha = 108^o, \beta = 120^o, \gamma = 120^o)$	$2\sin^{-1}\left(\frac{2\sqrt{605+184\sqrt{5}}-\sqrt{170-2\sqrt{5}}}{36\sqrt{5}}\right) + 4\sin^{-1}\left(\frac{\sqrt{58+18\sqrt{5}}-2}{12}\right) \approx 4.248741371\ sr$
6	Cuboctahedron	$4\sin^{-1}\left(\frac{1}{\sqrt{3}}\right) \approx 2.461918835\ sr$
7	Icosidodecahedron	$4\sin^{-1}\left(\sqrt{\frac{5+2\sqrt{5}}{15}}\right) \approx 3.673752748\ sr$

8	Small rhombicuboctahedron	$2\sin^{-1}\left(\frac{\sqrt{29-2\sqrt{2}}-\sqrt{9-4\sqrt{2}}}{4\sqrt{6}}\right)+6\sin^{-1}\left(\frac{\sqrt{5+2\sqrt{2}}-1}{4}\right)\approx 3.481429563\ sr$
9	Small rhombicosidodecahedron	$2\sin^{-1}\left(\frac{3\left(\sqrt{35+9\sqrt{5}}-\sqrt{5-\sqrt{5}}\right)}{20\sqrt{2}}\right)+2\sin^{-1}\left(\frac{\sqrt{355-20\sqrt{5}}-\sqrt{105-40\sqrt{5}}}{20\sqrt{3}}\right)+4\sin^{-1}\left(\frac{\sqrt{55+20\sqrt{5}}-\sqrt{5}}{10\sqrt{2}}\right)\approx 4.446308933\ sr$
10	Snub cube	$2\sin^{-1}\left(\frac{\left(1-\sqrt{1-K^2}\right)\sqrt{2K^2-1}}{K^2\sqrt{2}}\right)+8\sin^{-1}\left(\frac{\left(1-\sqrt{1-K^2}\right)\sqrt{4K^2-1}}{2K^2\sqrt{3}}\right)\approx 3.589629551\ sr$ $where, K\approx 0.928191378$
11	Snub dodecahedron	$2\sin^{-1}\left(\frac{\left(\sqrt{5}+1\right)}{4K}\right)-2\sin^{-1}\left(\sqrt{\left(\frac{5+2\sqrt{5}}{5}\right)}\left(\frac{\sqrt{1-K^2}}{K}\right)\right)+8\sin^{-1}\left(\frac{\left(1-\sqrt{1-K^2}\right)\sqrt{4K^2-1}}{2K^2\sqrt{3}}\right)\approx 4.509685356\ sr$ $where, K\approx 0.97273285$
12	Great rhombicuboctahedron $(\alpha=90^o,\beta=120^o,\gamma=135^o)$	$2\sin^{-1}\left(\frac{\sqrt{69+2\sqrt{2}}-\sqrt{9-4\sqrt{2}}}{12}\right)+2\sin^{-1}\left(\frac{\sqrt{26+12\sqrt{2}}-\sqrt{2}}{8}\right)+2\sin^{-1}\left(\frac{\sqrt{118+47\sqrt{2}}-\sqrt{10-\sqrt{2}}}{12\sqrt{2}}\right)=\frac{5\pi}{4}\approx 3.926990817\ sr$
13	Great rhombicosidodecahedron $(\alpha=90^o,\beta=120^o,\gamma=144^o)$	$2\sin^{-1}\left(\frac{\sqrt{171+4\sqrt{5}}-\sqrt{21-8\sqrt{5}}}{6\sqrt{10}}\right)+2\sin^{-1}\left(\frac{\sqrt{155+60\sqrt{5}}-\sqrt{5}}{20}\right)+2\sin^{-1}\left(\frac{\sqrt{165+25\sqrt{5}}-\sqrt{15-5\sqrt{5}}}{12\sqrt{2}}\right)=\frac{3\pi}{2}\approx 4.71238898\ sr$

From the above table of the solid angles, it is clear that the solid angle subtended by Great Rhombicosidodecahedron i.e. the largest Archimedean solid at its vertex is greatest $\approx \mathbf{4.71238898}\ sr$

Note: *Above articles had been derived & illustrated by Mr H.C. Rajpoot (B Tech, Mechanical Engineering)*

M.M.M. University of Technology, Gorakhpur-273010 (UP) India *April, 2015*

Email: rajpootharishchandra@gmail.com

Author's Home Page: https://notionpress.com/author/HarishChandraRajpoot