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HCR's Rank or Series Formula

(Logical-formula on Linear Permutations in Algebra)

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Abstract: A logistic was derived by Mr H.C. Rajpoot in March, 2010 which was formulated in a generalised form by him in Feb, 2014 by using some arbitrary terms to deal with complex problems of linear permutations. It is used to find out the correct order (called rank) of any randomly selected (or a given) linear permutation (like words, numbers & all other linear permutations) from a set of all the linear permutations arranged in a correct order (sequence). It is an expansion (series) formula of which each term corresponds to a certain article of any linear permutation. It is also applicable to position the linear permutations in correct order provided that the articles have at least one easily distinguishable property like appearance in shape, size, colour, surface-design etc. &all are equally significant at all the places (positions) in all possible linear arrangements. For ease of understanding, for a given alphabetic word or a positive integral number with 'n' letters or non-zero digits respectively, it is expressed as follows

$$R(word\ or\ number) = \sum_{i=1}^{i=n} F_i\left(\frac{P_i}{S_i}\right)$$

It has three new arbitrary parameters as $Formerity(F), Permuty(P)\&Similarity\ (S)$ which are expanded in a finite series. The values of these parameters depend on former (leading) articles, permutations of successive articles &repeatability of article, &The number of terms in that series is equal to total no. of articles (like letters, digits etc.) in any linear permutation (like word, number etc.).

Keywords: Linear permutation, $\mathbf{Formerity(F), Permuty(P)\&Similarity(S)}$.

1. INTRODUCTION

A number of linear permutations are obtained by permuting certain articles together. For ease of understanding & analysis, alphabetic words & the numbers are the best examples of linear permutations. Moreover, we are much more familiar with the linear arrangements of letters & digits in the correct sequences. Now, it is very well known fact that a number of alphabetic words are obtained by permuting all the letters (repetitive & non-repetitive) of a given alphabetic word. When all these words are arranged in the correct alphabetic order then each of the words has a certain alphabetic order (say rank) in all the permutations. But it is very difficult to find out the correct alphabetic order of any randomly selected the word in all its permutations.

Similarly, many a positive integral numbers are obtained by permuting all the non-zero digits (repetitive & non-repetitive) of a given positive integral number. When all these numbers are arranged in the correct increasing or decreasing order, each of the numbers has a certain numeric order (say rank) in all the permutations. But it is very difficult to find out the correct numeric order of any of the numbers in all its permutations.

A formula has been proposed based on logistics, which is applicable to find out the alphabetic order of a given word & numeric order (increasing or decreasing) of any positive integral number having non-zero digits.

2. HCR'S RANK OR SERIES FORMULA

We will study the randomly selected linear permutations, to find out their respective ranks (correct positions) in the group of all linear permutations arranged in correct order, in the following order:

1. Alphabeticwords
2. Positive integral numbers with only non-zero digits

3. Positive integral numbers with both zero & non-zero digits
4. Linear permutations of certain articles which are similar & dissimilar in shape, size, colour, surface-design etc. & are equally significant at all the positions in the linear arrangements.

2.1WORD SERIES

A certain number of the alphabetic words can be obtained by permuting all the letters in different sequences of alphabets. All the words of a series have identical letters but in different sequence of alphabet.

A correct alphabetic order of all the words obtained by permuting all the letters together of a given word is called word series of that word.

If a given word has n no. of the letters, out of which no. of the repetitive letters are

$p, q, r, s, \ldots$..then total no. of words (N_w) obtained by permuting all the letters is given as

$$N_w = \frac{n!}{p!\,q!\,r!\,s!\ldots}$$

N_w denotes the total no. of the words in word series of a given word.

Ex: Word 'TATA' has total no. of letters $n = 4$

No. of the repetitive letters, $p = 2\ (letter\ 'A')\&q = 2\ (letter'T')$

Total no. of the words is given as

$$N_w = \frac{4!}{2!\,2!} = \frac{4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1} = 6$$

The first & last words of the series are obtained by arranging the letters in certain alphabetic order as follows

$$firstword \rightarrow AATT \qquad (arrangingalltheletters in alphabetic order)$$

$$lastword \rightarrow TTAA \quad (arrangingalltheletters in reverse alphabetic order)$$

Thus, all the words can be arranged in a correct alphabetic order (rank) as follows

Word	Alphabetic Order (Rank)
AATT	1
ATAT	2
ATTA	3
TAAT	4
TATA*	5
TTAA	6

It's clear from the above word series that the given word TATA lies at fifth place in alphabetic order i.e. alphabetic order (rank) of TATA is 5 in its word series.

2.2 RANK OF WORD

Each of the words has a certain alphabetic order in its word series. Thus

"Correct alphabetic order of a given word in its word series is called Rank of that word''. It is denoted by $'R(word)'$.

Before proceeding further, let's first know the terminology related to rank of a given word

Let all the letters of a given word, keeping similar (repetitive) ones together in a linear-sequence be arranged in the correct alphabetic order. Now select & label the letter, in the alphabetic arrangementwhich is similar to the left-most letter in the given word. Now find the following parametric values of selected (labelled) letter

$\boldsymbol{Formerity(F)}$: $Formerity$of the selected letter is the total no. of letters dissimilar to it & appearing beforeit (i.e. lying to the left of it) in the correct alphabetic order of all theletters.

$\boldsymbol{Similarity(S)}$:Similarity of the selected letter is the total no. of the letters similar to it, including itself, in the alphabetic order of all the letters.

$\boldsymbol{Permuty(P)}$: $Permuty$of the selected letter is the total no. of permutations obtained all the letters, excluding selected one, in the alphabetic order. After this we find the following complex value of selected one

$\boldsymbol{PermutationValue(P_V)}$: $Permutationvalue$of any of the letters is given as follows

$$\Rightarrow P_V = F\left(\frac{P}{S}\right)$$

After finding this value of the selected letter, which is similar to left most of letter of given word, we cancel or eliminate or remove it from the alphabetic order & use the alphabetic order of remaining letters for next process of selection.

Further we select& label another letter, in the alphabetic order (after cancelling previous selected one), which is similar to the next or second left-most letter in the given word. Find its parametric values by the above definitions & then parametric value. Then cancel it from alphabetic order& select another letter similar to thenext(thirdleft-most)letter in the given word. Thus follow this cancellation method until reaches the last letter of a given or a randomly selected word& find the permutation values of all the letters in the given word. Process, of finding the parametric values F, P & S of all the letters, always starts from the left most (first) letter of a given or a selected word. Once all the letters are arranged in the correct alphabetic order then all the letters, according to the sequence of given word, are selected & eliminated one by one from alphabetic arrangement in order to find parametric values (F, P, S) of all the letters.

Mathematically, rank of a given word is the sum of permutation values of its all the letters

If a given word has total 'n' no. of the letters then its rank (R) is given as

$$R(word) = (P_V)_1 + (P_V)_2 + (P_V)_3 + (P_V)_4 + \dots\dots\dots\dots\dots\dots\dots .. + (P_V)_{n-1} + (P_V)_n$$

$$\Rightarrow R(word) = \sum_{i=1}^{i=n} (P_V)_i$$

On setting the value of P_V in the above equation, we have

$$R(word) = \sum_{i=1}^{i=n} \left(F\left(\frac{P}{S}\right)\right)_i = \sum_{i=1}^{i=n} F_i\left(\frac{P_i}{S_i}\right)$$

$$\Rightarrow \left[R(word) = \sum_{i=1}^{i=n} F_i\left(\frac{P_i}{S_i}\right)\right] \dots\dots\dots\dots\dots\dots\dots .. (I)$$

Note: Above formula is named as **HCR's Rank Formula.** This formula is equally applicable all the linear permutations like words, numbers etc. For finding out the permutation value of any selected letter/digit in a given word, note the following points

1. Permutation value of each of the letters/digits is always non-negative integer
2. Permutation value of last letter/digit is always 1.
3. Similarity of any non-repetitive letter/digit is always 1.

4. Permutation value is zero $iff Formerity\ (F)$ of a selected letter/digit is zero
5. The values of $Similarity(S)\&Permuty\ (P)$ are always positive integers.

2.3 WORKING STEPS (METHOD OF CANCELLATION)

Step 1: Arrange all the letters (repetitive & non-repetitive) of a given word in the correct alphabetic order placing the repetitive letters together in linear sequence.

Step 2: Find out the values of $Formerity(F),\ Similarity(S)\&Permuty\ (P)$& thus permutation value of each of the letters of given word from first (left most) letter up to last (right most) letter using correct alphabetic order of letters.

Step 3: Remove the selected letter, from alphabetic order of which permutation value (P_V) has been found out, for next letter of given word. Find out the permutation value (P_V) of next letter and similarly remove it from alphabetic order.

Repeat this process until the last letter is left in alphabetic order for which $P_V = 1$

Step 4: Add permutation values of all the letters to find out the rank of that word.

Note: Select each letter, from left most, according to given (original) word & label the same letter in their alphabetic order I.e. All the letters are selected one-by-one from their alphabetic order according to the arrangement of letters in given (original) word and labelled in their alphabetic order to find out the values of $Formerity(F),\ Similarity(S)\&Permuty\ (P)$ of particular selected letter according to their definitions.

2.4 ILLUSTRATIVE EXAMPLES

1. Let's consider the above word 'TATA' to find out its rank in its word series

Step 1: There are four letters in 'TATA' which are arranged, keeping repetitive letters together, in correct alphabetic order as follows

$$A \to A \to T * \to T$$

Step 2: According to given word 'TATA', first letter is 'T'. Hence select letter 'T' (as labelled) from above alphabetic order to find out the values of $Formerity(F),\ Similarity(S)\&Permuty\ (P)$ of selected letter 'T' of 'TATA' & thus permutation value as follows

$$\Rightarrow F_1 = No. of letters dissimilar \& appearing before selected letter\ 'T' of TATA in alphabetic order$$

$$= 2 \quad (two letters\ 'A'\&\ 'A' are appearing before\ 'T' which are dissimilar to\ 'T')$$

$$\Rightarrow S_1 = No. of letters similar to selected letter\ 'T' of TATA in above alphabetic order,$$

$$including itself$$

$$= 1 + 1 = 2 \quad (there is only one letter\ 'T' which is similar to selected letter 'T')$$

$$\Rightarrow P_1 = No. of permutations obtained from remaining letters\ 'A',\ 'A'\&'T'$$

$$(excluding selected 'T' of TATA) in above alphabetic order$$

$$= \frac{3!}{2!} = \frac{6}{2} = 3$$

$$\therefore permutationvalueoffirstselectedletter\ \ 'T',\ \ (P_V)_1 = F_1\left(\frac{P_1}{S_1}\right)$$

$$= 3\left(\frac{2}{2}\right) = 3$$

Step 3: Since, the permutation value of one of the letters 'T' has been determined thus remove it from alphabetic order. Hence, alphabetic order of remaining letters is

$$A* \rightarrow A \rightarrow T$$

According to given word 'TATA', second (next) letter is 'A'. Hence select letter 'A' (as labelled) from above alphabetic order to find out the values of $Formerity(F),\ Similarity(S)\&Permuty\ (P)$ of selected letter 'A' of 'TATA' & thus permutation value as follows

$$\Rightarrow F_2 = No.oflettersdissimilar\&appearingbeforeselectedletter\ \ 'A'ofTATAinalphabeticorder$$

$$= 0 \qquad (thereisnoletterdissimilarto'A'\&appearingbefore\ \ 'A')$$

$$\therefore permutationvalueofselectedletter\ \ 'A',\ \ (P_V)_2 = F_2\left(\frac{P_2}{S_2}\right) = 0$$

(In this case the values of $S\&P$ need not be determined since $F = 0$)

Step 4: Since, the permutation value of one of the letters 'A' has been determined thus remove it from above alphabetic order. Hence, alphabetic order of remaining letters is

$$A \rightarrow T*$$

According to given word 'TATA', third (next) letter is 'T'. Hence select letter 'T' (as labelled) from above alphabetic order to find out the values of $Formerity(F),\ Similarity(S)\&Permuty\ (P)$ of selected letter 'T' of 'TATA' & thus permutation value as follows

$$\Rightarrow F_3 = No.ofdissimilarlettersappearingbeforeselectedletter\ \ 'T'ofTATAinalphabeticorder$$

$$= 1 \qquad (thereisonlyoneletter\ \ 'A'dissimilar\&appearingbefore\ \ 'T')$$

$$\Rightarrow S_3 = No.oflettersimilartoselectedletter\ \ 'T'ofTATAinabovealphabeticorder,$$

$$includingitself$$

$$= 1 \qquad (thereisonlyoneletter\ \ 'T'whichissimilartoitself)$$

$$\Rightarrow P_3 = No.ofpermutationsobtainedfromremaininglettersthatis\ \ 'A'$$

$$(excludingselected'T'ofTATA)inabovealphabeticorder$$

$$= 1! = 1$$

$$\therefore permutationvalueofselectedletter\ \ 'T',\ \ (P_V)_3 = F_3\left(\frac{P_3}{S_3}\right)$$

$$= 1\left(\frac{1}{1}\right) = 1$$

Step 5: Since the permutation value of selected letter 'T' has been determined thus remove it from alphabetic order. Hence, alphabetic order of remaining letters is

$$A\,*$$

Since, 'A' is the fourth & last letter (as labelled) of the word TATA hence its permutation value is 1

$$\Rightarrow (P_V)_4 = 1$$

Thus, rank of 'TATA' is given as follows

$$\Rightarrow R(TATA) = (P_V)_1 + (P_V)_2 + (P_V)_3 + (P_V)_4$$

$$= 3 + 0 + 1 + 1 = 5$$

Similarly, rank of TAAT can be determined following the above procedure as follows

Arrange all the letters in alphabetic order as follows

$$A \rightarrow A \rightarrow T \rightarrow T$$

Now, according to the given word 'TAAT' select letter one by one from left to find their respective permutation values & add all the values as follows

$$\Rightarrow R(TAAT) = (P_V)_1 + (P_V)_2 + (P_V)_3 + (P_V)_4$$

$$= F_1\left(\frac{P_1}{S_1}\right) + F_2\left(\frac{P_2}{S_2}\right) + F_3\left(\frac{P_3}{S_3}\right) + F_4\left(\frac{P_4}{S_4}\right)$$

$$= 2\left(\frac{\left(\frac{3!}{2!}\right)}{2}\right) + 0\left(\frac{2!}{2}\right) + 0\left(\frac{1!}{1}\right) + 1 = 3 + 1 = 4$$

Similarly, rank of 'TTAA'

$$\Rightarrow R(TTAA) = (P_V)_1 + (P_V)_2 + (P_V)_3 + (P_V)_4$$

$$= F_1\left(\frac{P_1}{S_1}\right) + F_2\left(\frac{P_2}{S_2}\right) + F_3\left(\frac{P_3}{S_3}\right) + F_4\left(\frac{P_4}{S_4}\right)$$

$$= 2\left(\frac{\left(\frac{3!}{2!}\right)}{2}\right) + 2\left(\frac{\left(\frac{2!}{2!}\right)}{1}\right) + 0\left(\frac{1!}{1}\right) + 1 = 3 + 2 + 1 = 6$$

It is obvious that the above formula is extremely useful for finding out the ranks of words having greater no. of letters (repetitive or non-repetitive) also for finding ranks of numbers having greater no. of digits.

2. Let's find out the rank of word '**DISSOCIATE**' in its word series

Step 1: There total ten letters in 'DISSOCIATE' which are arranged, keeping repetitive letters together, in correct alphabetic order as follows

$$A \rightarrow C \rightarrow D * \rightarrow E \rightarrow I \rightarrow I \rightarrow O \rightarrow S \rightarrow S \rightarrow T$$

Step 2: According to given word 'DISSOCIATE', first letter is 'D'. Hence select letter 'D' (as labelled) from alphabetic order to find out the values of $Formerity(F),\ Similarity(S)\&Permuty\ (P)$ of selected letter 'D' of 'DISSOCIATE' & thus permutation value as follows

$$\Rightarrow F_1 = No. of dissimilar letters appearing before first letter\ 'D' of DISSOCIATE in alphabetic order$$

$$= 2 \quad (two letters\ 'A'\&\ 'C' are appearing before\ 'D' which are dissimilar to\ 'D')$$

$$\Rightarrow S_1 = No. of letters similar to first selected letter\ 'D' of DISSOCIATE in alphabetic order, including itself$$

$$= 1 \quad (there is only one letter\ 'D' which is similar to itself\ 'D')$$

$$\Rightarrow P_1 = No. of permutations obtained from remaining letters in alphabetic order which are A, C, E, I, I, O, S, S, T (excluding selected 'D' of DISSOCIATE) in alphabetic order$$

$$= \frac{9!}{2!\,2!} = \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1} = \frac{362880}{4} = 90720$$

$$\therefore permutation value of first selected letter\ 'T',\ \ (P_V)_1 = F_1\left(\frac{P_1}{S_1}\right)$$

$$= 2\left(\frac{90720}{1}\right) = 181440$$

Step 3: Since, the permutation value of the selected letter 'D' has been determined thus remove it from alphabetic order. Hence, alphabetic order of remaining letters is

$$A \to C \to E \to I* \to I \to O \to S \to S \to T$$

Now, according to given word 'DISSOCIATE', second (next) letter is 'I'. Hence select letter 'I' (as labelled) from alphabet order to find out the values of $Formerity\ (F),\ Similarity\ (S)\&\ Permuty\ (P)$ of selected letter 'I' of 'DISSOCIATE' & thus permutation value as follows

$$\Rightarrow F_2 = No. of\ dissimilar\ letters\ appearing\ before\ selected\ letter\ \ 'I'\ in\ alphabetic\ order$$

$$= 3 \quad (there\ are\ three\ letters\ A, C, \&\ E\ dissimilar\ to\ 'I'\&\ appearing\ before\ \ 'I')$$

$$\Rightarrow S_2 = No. of\ letters\ similar\ to\ selected\ letter\ \ 'I', from\ DISSOCIATE, in\ above\ alphabetic\ order, including\ itself\ (labelled\ 'I')$$

$$= 2 \quad (there\ are\ two\ letters\ I, I\ including\ itself\ (selected\ I)\ )$$

$$\Rightarrow P_2 = No. of\ permutations\ obtained\ from\ remaining\ letters\ in\ alphabetic\ order\ which\ are\ A, C, E, I, O, S, S, T\ (\ excluding\ selected\ letter\ 'I')\ in\ above\ alphabetic\ order$$

$$= \frac{8!}{2!} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = \frac{40320}{2} = 20160$$

$$\therefore \text{permutation value of second selected letter } 'A', \ (P_V)_2 = F_2\left(\frac{P_2}{S_2}\right) = 3\left(\frac{20160}{2}\right) = 30240$$

Step 4: Since, the permutation value of one of the letters 'I' has been determined thus remove it from alphabetic order. Hence, alphabetic order of remaining letters is

$$A \to C \to E \to I \to O \to S * \to S \to T$$

According to given word 'DISSOCIATE', third (next) letter is 'S'. Hence select letter 'S' (as labelled) from above alphabetic order to find out the values of $Formerity\ (F),\ Similarity\ (S)\&\ Permuty\ (P)$ of selected letter 'I' of 'DISSOCIATE' & thus permutation value as follows

$$\Rightarrow F_3 = No.of\ \ letters\ dissimilar\ \&\ appearing\ before\ selected\ letter\ 'S'\ of\ DISSOCIATE$$

$$in\ above\ alphabetic\ order$$

$$= 5 \quad (there\ are\ five\ letters\ A, C, E, I, O\ \ dissimilar\ \&\ appearing\ before\ labelled\ 'S')$$

$$\Rightarrow S_3 = No.of\ letters\ similar\ to\ selected\ letter\ 'S'\ in\ above\ alphabetic\ order,$$

$$including\ itself$$

$$= 2 \qquad (there\ are\ two\ similar\ letters\ S, S\ including\ itself)$$

$$\Rightarrow P_3 = No.of\ permutations\ obtained\ from\ remaining\ letters\ A, C, E, I, O, S, T$$

$$(\ excluding\ selected\ 'S')\ in\ above\ alphabetic\ order$$

$$= 7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$$

$$\therefore permutation\ value\ of\ selected\ letter\ 'S',\ (P_V)_3 = F_3\left(\frac{P_3}{S_3}\right)$$

$$= 5\left(\frac{5040}{2}\right) = 12600$$

Step 5: Since the permutation value of selected letter 'S' has been determined thus remove it i.e. one of the repetitive letters 'S' from alphabetic order. Hence, alphabetic order of remaining letters is

$$A \to C \to E \to I \to O \to S * \to T$$

According to given word 'DISSOCIATE', next letter is 'S'. Hence select letter 'S' (as labelled) from above alphabetic order to find out the values of $Formerity\ (F),\ Similarity\ (S)\&\ Permuty\ (P)$ of selected letter 'S' of 'DISSOCIATE' & thus permutation value as follows

$$\Rightarrow F_4 = No.of\ \ letters\ dissimilar\ \&\ appearing\ before\ selected\ letter\ 'S'\ of\ DISSOCIATE$$

$$in\ above\ alphabetic\ order$$

$$= 5 \quad (there\ are\ five\ letters\ A, C, E, I, O\ \ dissimilar\ \&appearing\ before\ labelled\ 'S')$$

$$\Rightarrow S_4 = No.of\ letters\ similar\ to\ selected\ letter\ 'S'\ in\ above\ alphabetic\ order,$$

$$including\ itself$$

$$= 1 \qquad (there\ is\ only\ one\ letter\ S\ similar\ to\ itself)$$

$$\Rightarrow P_4 = No. of\ permutations\ obtained\ from\ remaining\ letters\ A, C, E, I, O, T$$

$$(\ excluding\ selected\ 'S'\ )\ in\ above\ alphabetic\ order$$

$$= 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$$

$$\therefore permutation\ value\ of\ selected\ letter\ {}'S',\ (P_V)_4 = F_4 \left(\frac{P_4}{S_4}\right)$$

$$= 5\left(\frac{720}{1}\right) = 3600$$

Step 6: Since the permutation value of selected letter 'S' has been determined thus remove it from alphabetic order. Hence, alphabetic order of remaining letters is

$$A \to C \to E \to I \to O * \to T$$

According to given word 'DISSOCIATE', next letter is 'O'. Hence select letter 'O' (as labelled) from above alphabetic order to find out the values of $Formerity\ (F),\ Similarity\ (S) \&\ Permuty\ (P)$ of selected letter 'O' of 'DISSOCIATE' & thus permutation value as follows

$$\Rightarrow F_5 = No. of\ \ letters\ dissimilar\ \&\ appearing\ before\ selected\ letter\ {}'S'\ of\ DISSOCIATE$$

$$in\ above\ alphabetic\ order$$

$$= 4 \quad (there\ are\ four\ letters\ A, C, E, I\ \ dissimilar\ \&\ apearing\ before\ labelled\ {}'O')$$

$$\Rightarrow S_5 = No. of\ letters\ similar\ to\ selected\ letter\ {}'O'\ in\ above\ alphabetic\ order,$$

$$including\ itself$$

$$= 1 \qquad (there\ is\ only\ one\ letter\ O\ similar\ to\ itself)$$

$$\Rightarrow P_5 = No. of\ permutations\ obtained\ from\ remaining\ letters\ A, C, E, I, T$$

$$(\ excluding\ selected\ 'O'\ )\ in\ above\ alphabetic\ order$$

$$= 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

$$\therefore permutation\ value\ of\ selected\ letter\ {}'O',\ (P_V)_5 = F_5 \left(\frac{P_5}{S_5}\right)$$

$$= 4\left(\frac{120}{1}\right) = 480$$

Step 7: Since the permutation value of selected letter 'O' has been determined thus remove it from alphabetic order. Hence, alphabetic order of remaining letters is

$$A \to C * \to E \to I \to T$$

According to given word 'DISSOCIATE', next letter is 'C'. Hence select letter 'C' (as labelled) from above alphabetic order to find out the values of $Formerity\ (F),\ Similarity\ (S) \&\ Permuty\ (P)$ of selected letter 'C' of 'DISSOCIATE' & thus permutation value as follows

$$\Rightarrow F_6 = No. of\ \ letters\ dissimilar\ \&\ appearing\ before\ selected\ letter\ {}'C'\ of\ DISSOCIATE$$

$$in\ above\ alphabetic\ order$$

$$= 1 \quad (there\ is\ only\ one\ letter\ A\ \ dissimilar\ \&\ apearing\ before\ labelled\ 'C')$$

$$\Rightarrow S_6 = No.of\ letters\ similar\ to\ selected\ letter\ 'C'\ in\ above\ alphabetic\ order,$$

$$including\ itself$$

$$= 1 \qquad (there\ is\ only\ one\ letter\ C\ similar\ to\ itself)$$

$$\Rightarrow P_6 = No.of\ permutations\ obtained\ from\ remaining\ letters\ A, E, I, T$$

$$(\ excluding\ selected\ 'C'\ )\ in\ above\ alphabetic\ order$$

$$= 4! = 4 \times 3 \times 2 \times 1 = 24$$

$$\therefore permutation\ value\ of\ selected\ letter\ 'C',\ (P_V)_6 = F_6\left(\frac{P_6}{S_6}\right)$$

$$= 1\left(\frac{24}{1}\right) = 24$$

Step 8: Since the permutation value of selected letter 'C' has been determined thus remove it from alphabetic order. Hence, alphabetic order of remaining letters is

$$A \rightarrow E \rightarrow I * \rightarrow T$$

According to given word 'DISSOCIATE', next letter is 'I'. Hence select letter 'I' (as labelled) from above alphabetic order to find out the values of $Formerity\ (F),\ Similarity\ (S) \&\ Permuty\ (P)$ of selected letter 'I' of 'DISSOCIATE' & thus permutation value as follows

$$\Rightarrow F_7 = No.of\ \ letters\ dissimilar\ \&\ appearing\ before\ selected\ letter\ 'I'\ of\ DISSOCIATE$$

$$in\ above\ alphabetic\ order$$

$$= 2 \quad (there\ are\ two\ letter\ A, E\ \ dissimilar\ \&\ apearing\ before\ labelled\ 'I')$$

$$\Rightarrow S_7 = No.of\ letters\ similar\ to\ selected\ letter\ 'I'\ in\ above\ alphabetic\ order,$$

$$including\ itself$$

$$= 1 \qquad (there\ is\ only\ one\ letter\ I\ similar\ to\ itself)$$

$$\Rightarrow P_7 = No.of\ permutations\ obtained\ from\ remaining\ letters\ A, E, T$$

$$(\ excluding\ selected\ 'I'\ )\ in\ above\ alphabetic\ order$$

$$= 3! = 3 \times 2 \times 1 = 6$$

$$\therefore permutation\ value\ of\ selected\ letter\ 'C',\ (P_V)_7 = F_7\left(\frac{P_7}{S_7}\right)$$

$$= 2\left(\frac{6}{1}\right) = 12$$

Step 9: Since the permutation value of selected letter 'I' has been determined thus remove it from alphabetic order. Hence, alphabetic order of remaining letters is

$$A * \rightarrow E \rightarrow T$$

According to given word 'DISSOCIATE', next letter is 'A'. Hence select letter 'A' (as labelled) from above alphabetic order to find out the values of $Formerity\ (F),\ Similarity\ (S) \&\ Permuty\ (P)$ of selected letter 'A' of 'DISSOCIATE' & thus permutation value as follows

$$\Rightarrow F_8 = No.of\ \ letters\ dissimilar\ \&\ appearing\ before\ selected\ letter\ 'A'\ of\ DISSOCIATE$$

$$in\ above\ alphabetic\ order$$

$$= 0 \quad (there\ is\ no\ letter\ \ dissimilar\ \&\ apearing\ before\ labelled\ 'A')$$

$$\therefore permutation\ value\ of\ selected\ letter\ 'A',\ (P_V)_8 = F_8\left(\frac{P_8}{S_8}\right)$$

$$= 0\left(\frac{P_8}{S_8}\right) = 0$$

Step 10: Since the permutation value of selected letter 'A' has been determined thus remove it from alphabetic order. Hence, alphabetic order of remaining letters is

$$E \rightarrow T *$$

According to given word 'DISSOCIATE', next letter is 'T'. Hence select letter 'T' (as labelled) from above alphabetic order to find out the values of $Formerity\ (F),\ Similarity\ (S) \&\ Permuty\ (P)$ of selected letter 'T' of 'DISSOCIATE' & thus permutation value as follows

$$\Rightarrow F_9 = No.of\ \ letters\ dissimilar\ \&\ appearing\ before\ selected\ letter\ 'T'\ of\ DISSOCIATE$$

$$in\ above\ alphabetic\ order$$

$$= 1 \quad (there\ is\ only\ one\ letter\ E\ \ dissimilar\ \&\ apearing\ before\ labelled\ 'T')$$

$$\Rightarrow S_9 = No.of\ letters\ similar\ to\ selected\ letter\ 'T'\ in\ above\ alphabetic\ order,$$

$$including\ itself$$

$$= 1 \qquad (there\ is\ only\ one\ letter\ T\ similar\ to\ itself)$$

$$\Rightarrow P_9 = No.of\ permutations\ obtained\ from\ remaining\ letters\ 'E'$$

$$(\ excluding\ selected\ 'T'\ )\ in\ above\ alphabetic\ order$$

$$= 1! = 1$$

$$\therefore permutation\ value\ of\ selected\ letter\ 'T',\ (P_V)_9 = F_9\left(\frac{P_9}{S_9}\right)$$

$$= 1\left(\frac{1}{1}\right) = 1$$

Step 11: Since the permutation value of selected letter 'T' has been determined thus remove it from alphabetic order. Hence, alphabetic order of remaining letters is

$$E*$$

According to given word 'DISSOCIATE', next & last letter is 'E'. Hence permutation value of last letter 'E' of given word 'DISSOCIATE'

$$\therefore permutation\ value\ of\ last\ letter\ 'E',\ (P_V)_{10} = 1$$

Thus, rank (R) of word 'DISSOCIATE' is given as follows

$$\Rightarrow R(DISSOCIATE)$$

$$= (P_V)_1 + (P_V)_2 + (P_V)_3 + (P_V)_4 + (P_V)_5 + (P_V)_6 + (P_V)_7 + (P_V)_8 + (P_V)_9 + (P_V)_{10}$$

$$= 181440 + 30240 + 12600 + 3600 + 480 + 24 + 12 + 0 + 1 + 1 = 228398$$

Total no. of the words in the word series of 'DISSOCIATE' is given as

$$= \frac{10!}{2!\,2!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1} = 907200$$

First & last word of series can be obtained as follows

$$first\ word \rightarrow ACDEIIOSST\ (by\ arranging\ in\ correct\ alphabetic\ order)$$

$$last\ word \rightarrow TSSOIIEDCA\ (by\ arranging\ in\ reverse\ alphabetic\ order)$$

Whole series of the word 'DISSOCIATE' along with its rank (R=228398) will be as follows

WORD	**RANK (R)**
ACDEIIOSST	1
ACDEIIOSTS	2
ACDEIIOTSS	3
ACDEIISOST	4
ACDEIISOTS	5
………………….. ………………….. …………………..	……………….. ……………….. ………………..
DISSOCIAET	228397
DISSOCIATE*	228398*
DISSOCIEAT	228399
………………… ………………… …………………	……………….. ……………….. ………………..
TSSOIIEADC	907196
TSSOIIECAD	907197
TSSOIIECDA	907198
TSSOIIEDAC	907199
TSSOIIEDCA	907200

In the above example, the steps are very long to explain but these can be performed in a single step of calculations. Let's find out the ranks of other words from above word series

Rank of 'TSSOIIEADC' is determined as follows

Arrange all the letters of 'TSSOIIEADC' in the correct alphabetic order as follows

$$A \to C \to D \to E \to I \to I \to O \to S \to S \to T$$

Now, select & remove the letters one by one from alphabetic order to find rank as follows

$$R(TSSOIIEADC) = 9\left(\frac{\left(\frac{9!}{2!2!}\right)}{1}\right) + 7\left(\frac{\left(\frac{8!}{2!}\right)}{2}\right) + 7\left(\frac{\left(\frac{7!}{2!}\right)}{1}\right) + 6\left(\frac{\left(\frac{6!}{2!}\right)}{1}\right) + 4\left(\frac{5!}{2}\right) + 4\left(\frac{4!}{1}\right)$$

$$+3\left(\frac{3!}{1}\right) + 0\left(\frac{2!}{1}\right) + 1\left(\frac{1!}{1}\right) + 1$$

$$= 816480 + 70560 + 17640 + 2160 + 240 + 96 + 18 + 0 + 1 + 1 = 907196$$

Above result is correct from the table of word series.

Similarly, rank of 'TSSOIIEDAC' can be determined using above alphabetic order as follows

$$\Rightarrow\ R(TSSOIIEDAC) = 9\left(\frac{\left(\frac{9!}{2!2!}\right)}{1}\right) + 7\left(\frac{\left(\frac{8!}{2!}\right)}{2}\right) + 7\left(\frac{\left(\frac{7!}{2!}\right)}{1}\right) + 6\left(\frac{\left(\frac{6!}{2!}\right)}{1}\right) + 4\left(\frac{5!}{2}\right) + 4\left(\frac{4!}{1}\right)$$

$$+3\left(\frac{3!}{1}\right) + 2\left(\frac{2!}{1}\right) + 0\left(\frac{1!}{1}\right) + 1$$

$$= 816480 + 70560 + 17640 + 2160 + 240 + 96 + 18 + 4 + 0 + 1 = 907199$$

Above result is correct from the table of word series.

Similarly, rank of last word 'TSSOIIEDAC' can be determined using above alphabetic order as follows

$$\Rightarrow\ R(TSSOIIEDCA) = 9\left(\frac{\left(\frac{9!}{2!2!}\right)}{1}\right) + 7\left(\frac{\left(\frac{8!}{2!}\right)}{2}\right) + 7\left(\frac{\left(\frac{7!}{2!}\right)}{1}\right) + 6\left(\frac{\left(\frac{6!}{2!}\right)}{1}\right) + 4\left(\frac{5!}{2}\right) + 4\left(\frac{4!}{1}\right)$$

$$+3\left(\frac{3!}{1}\right) + 2\left(\frac{2!}{1}\right) + 1\left(\frac{1!}{1}\right) + 1$$

$$= 816480 + 70560 + 17640 + 2160 + 240 + 96 + 18 + 4 + 1 + 1 = 907200$$

Above result is correct from the table of word series.

Note: Rank of last later also denotes the total no. of words in the series.

2.5 DIRECT APPLICATION OF HCR's FORMULA

Problem 1: Find out alphabetic order of word '**CALCULUS**' in the group of the words obtained by permuting all the letters together.

Sol.

Arrange all the letters of 'CALCULUS' in the correct alphabetic order, keeping repetitive letters together, as follows

$$A \to C \to C \to L \to L \to U \to U \to S$$

Now, select & remove the letters one by one from alphabetic order to find out values of $Formerity, Permuty$ & $Similarity$ of each letter & add their permutation values together (following the same procedure as mentioned in above illustrative examples)

Using HCR's Rank or Series Formula as follows

$$R(CALCULUS)$$

$$= F_1\left(\frac{P_1}{S_1}\right) + F_2\left(\frac{P_2}{S_2}\right) + F_3\left(\frac{P_3}{S_3}\right) + F_4\left(\frac{P_4}{S_4}\right) + F_5\left(\frac{P_5}{S_5}\right) + F_6\left(\frac{P_6}{S_6}\right) + F_7\left(\frac{P_7}{S_7}\right) + F_8\left(\frac{P_8}{S_8}\right)$$

$$= 1\left(\frac{\left(\frac{7!}{2!2!}\right)}{2}\right) + 0\left(\frac{\left(\frac{6!}{2!2!}\right)}{1}\right) + 1\left(\frac{\left(\frac{5!}{2!}\right)}{2}\right) + 0\left(\frac{\left(\frac{4!}{2!}\right)}{1}\right) + 1\left(\frac{3!}{2}\right) + 0\left(\frac{2!}{1}\right) + 0\left(\frac{1!}{1}\right) + 1$$

$$= 630 + 0 + 30 + 0 + 3 + 0 + 0 + 1 = 664$$

While total no. of the words in the group (Word Series) is given as follows

$$N_N = \frac{8!}{2!\,2!\,2!} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1 \times 2 \times 1} = 5040$$

Thus alphabetic order of 'CALCULUS' is 664 out of 5040 words.

Problem 2: Find out alphabetic order of word '**GEOMETRY**' in the group of the words obtained by permuting all the letters together.

Sol.

Arrange all the letters of 'GEOMETRY' in the correct alphabetic order, keeping repetitive letters together, as follows

$$E \to E \to G \to M \to O \to R \to T \to Y$$

Now, select & remove the letters one by one from alphabetic order to find out values of $Formerity, Permuty$ & $Similarity$ of each letter & add their permutation values together

Using HCR's Rank or Series Formula as follows

$$R(GEOMETRY)$$

$$= F_1\left(\frac{P_1}{S_1}\right) + F_2\left(\frac{P_2}{S_2}\right) + F_3\left(\frac{P_3}{S_3}\right) + F_4\left(\frac{P_4}{S_4}\right) + F_5\left(\frac{P_5}{S_5}\right) + F_6\left(\frac{P_6}{S_6}\right) + F_7\left(\frac{P_7}{S_7}\right) + F_8\left(\frac{P_8}{S_8}\right)$$

$$= 2\left(\frac{\left(\frac{7!}{2!}\right)}{1}\right) + 0\left(\frac{6!}{2}\right) + 2\left(\frac{5!}{1}\right) + 1\left(\frac{4!}{1}\right) + 0\left(\frac{3!}{1}\right) + 1\left(\frac{2!}{1}\right) + 0\left(\frac{1!}{1}\right) + 1$$

$$= 5040 + 0 + 240 + 24 + 0 + 2 + 0 + 1 = 5307$$

While total no. of the words in the group (Word Series) is given as follows

$$N_N = \frac{8!}{2!\,2!} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1} = 10080$$

Thus alphabetic order of 'GEOMETRY' is 5307 out of 10080 words.

Problem 3: Find out alphabetic order of word '**MATHEMATICS**' in the group of the words obtained by permuting all the letters together.

Sol.

Arrange all the letters of 'MATHEMATICS' in the correct alphabetic order, keeping repetitive letters together, as follows

$$A \to A \to C \to E \to H \to I \to M \to M \to S \to T \to T$$

Now, select & remove the letters one by one from alphabetic order to find out values of $Formerity, Permuty$ & $Similarity$ of each letter & add their permutation values together

Using HCR's Rank or Series Formula as follows

$$R(MATHEMATICS)$$

$$= F_1\left(\frac{P_1}{S_1}\right) + F_2\left(\frac{P_2}{S_2}\right) + F_3\left(\frac{P_3}{S_3}\right) + F_4\left(\frac{P_4}{S_4}\right) + F_5\left(\frac{P_5}{S_5}\right) + F_6\left(\frac{P_6}{S_6}\right) + F_7\left(\frac{P_7}{S_7}\right) + F_8\left(\frac{P_8}{S_8}\right) + F_9\left(\frac{P_9}{S_9}\right) + F_{10}\left(\frac{P_{10}}{S_{10}}\right) + F_{11}\left(\frac{P_{11}}{S_{11}}\right)$$

$$= 6\left(\frac{\left(\frac{10!}{2!2!}\right)}{2}\right) + 0\left(\frac{\left(\frac{9!}{2!}\right)}{2}\right) + 7\left(\frac{8!}{2}\right) + 3\left(\frac{7!}{1}\right) + 2\left(\frac{6!}{1}\right) + 3\left(\frac{5!}{1}\right) + 0\left(\frac{4!}{1}\right) + 3\left(\frac{3!}{1}\right) + 1\left(\frac{2!}{1}\right) + 0\left(\frac{1!}{1}\right) + 1$$

$$= 2721600 + 0 + 141120 + 15120 + 1440 + 360 + 0 + 18 + 2 + 0 + 1 = 2879661$$

While total no. of the words in the group (Word Series) is given as follows

$$N_N = \frac{11!}{2!\,2!\,2!} = \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1 \times 2 \times 1} = 4989600$$

Thus alphabetic order of 'MATHEMATICS' is 2879661 out of 4989600 words.

3. POSITIVE INTEGRAL NUMBERS WITH NON-ZERO DIGITS

3.1 NUMBER SERIES

A certain no. of numbers can be obtained by permuting all the non-zero digits together of a given number. These numbers can be arranged or grouped in increasing or decreasing order.

Thus, a group or series of all the numbers, arranged in increasing or decreasing order, obtained by permuting all the non-zero digits of a given number is called **Number Series**.

If a number has 'n' no. of non-zero digits out of which no. of repetitive digits are $p, q, r, s, \dots$ .... Then the total no. of the numbers formed

$$N_N = \frac{n!}{p!\,q!\,r!\,s!\dots}$$

N_w denotes the total no. of the numbers formed in the Number Series.

3.2 RANK OF POSITIVE INTEGRAL NUMBER

A given number has a certain increasing or decreasing order no. in its series that is called **Rank** of given number. It is denoted by symbol $R(Number)$.

Rank (R) of a given number having 'n' no. of non-zero digits is given by **HCR's Rank Formula** which is used for the words.

$$\Rightarrow \left[\quad R(number) = \sum_{i=1}^{i=n} F_i\left(\frac{P_i}{S_i}\right)\right] \qquad \dots\dots\dots\dots\dots\dots\dots(I)$$

Note: All the symbols have their usual meanings as in case of word series**.** If the rank of a given number is taken in increasing order it is denoted by symbol "$R(number \uparrow)$" $\quad [\,'\uparrow'\ signifies\ increasing\ order]$

Similarly, if the rank of a given number is taken in decreasing order it is denoted by the symbol
"$R(number \downarrow)$" $[' \downarrow '\ signifies\ decreasing\ order]$

3.3 WORKING STEPS

Step 1: Arrange all the non-zero digits, keeping repetitive digits together, in increasing or decreasing order according to the requirement/problem.

Step 2: Find the permutation value of each of the digits of the given number using required order of digits by following the same procedure as mentioned above for a given word.

Add the permutation values of all the digits to find rank of the given number in increasing or decreasing order.

Note: Select each digit, from left most, according to given (original) number & label the same digit in their numeric order I.e. All the digits are selected one-by-one from their numeric order (increasing or decreasing) according to the arrangement of digits in the given (original) number and labelled in numeric order to find values of $Formerity\ (F)$, $Similarity\ (S)$& $Permuty\ (P)$ of particular selected digit according to their definitions.

For avoiding the difficulty in finding rank in decreasing order let's co-relate them

Ranks of a given number in increasing and decreasing orders can be correlated as

$$R(number \downarrow) = N_N - R(number \uparrow) + 1$$

Where, N_N denotes total no. of the numbers formed in the series of a given number.

It is easier to find out the rank in increasing order rather than in decreasing order following the same procedure as followed for a given word.

3.4 ILLUSTRATIVE EXAMPLES ON POSITIVE INTEGRAL NUMBER WITH NON-ZERO DIGITS

Example 1: Let's find out the rank of a number 5252 in its series in increasing order

Step 1: Arrange all the non-zero digits of 5252, keeping repetitive digits together, in increasing order as follows

$$2 \rightarrow 2 \rightarrow 5 * \rightarrow 5$$

Step 2: According to given (original) number '5252', first digit is '5'. Hence select letter '5' (as labelled) from above increasing order to find out the values of $Formerity\ (F)$, $Similarity\ (S)$& $Permuty\ (P)$ of selected digit '5' of '5252' & thus permutation value as follows

$$\Rightarrow F_1 = No.of\ digits\ dissimilar\ \&\ appearing\ before\ selected\ digit\ '5'of\ 5252\ in\ increasing\ order$$

$$= 2 \quad (two\ digits\ '2'\&\ '2'\ are\ appearing\ before\ '5'\ which\ are\ dissimilar\ to\ '5')$$

$$\Rightarrow S_1 = No.of\ digits\ similar\ to\ selected\ digit\ '5'\ of\ 5252\ in\ above\ increasing\ order,$$

$$including\ itself$$

$$= 1 + 1 = 2 \quad (there\ are\ two\ digits\ 5, 5\ similar\ to\ each\ other\ including\ labelled\ '5')$$

$$\Rightarrow P_1 = No.of\ permutations\ obtained\ from\ remaining\ digits\ '2',\ '2'\&\ '5'$$

$$(\ excluding\ labelled/selected\ '5'\ )\ in\ above\ numeric\ order$$

$$= \frac{3!}{2!} = \frac{6}{2} = 3$$

$$\therefore permutation\ value\ of\ first\ selected\ digit\ '5',\ (P_V)_1 = F_1\left(\frac{P_1}{S_1}\right)$$

$$= 3\left(\frac{2}{2}\right) = 3$$

Step 3: Since, the permutation value the selected digit '5' has been determined thus remove it from above numeric order. Hence, increasing numeric order of remaining digits is

$$2 * \rightarrow 2 \rightarrow 5$$

According to given number '5252', second (next) digit is '2'. Hence select digit '2' (as labelled) from above numeric order to find out the values of $Formerity\ (F),\ Similarity\ (S) \&\ Permuty\ (P)$ of selected digit '2' of '5252' & thus permutation value as follows

$$\Rightarrow F_2 = No.of\ \ digits\ dissimilar\ \&\ appearing\ before\ selected\ digit\ '2' of\ 5252\ in\ alphabetic\ order$$

$$= 0 \qquad (there\ is\ no\ digit\ dissimilar\ \&\ appearing\ before\ '2')$$

$$\therefore permutation\ value\ of\ selected\ digit\ '2',\ (P_V)_2 = F_2\left(\frac{P_2}{S_2}\right) = 0$$

(In this case the values of S & P need not be determined since $F = 0$)

Step 4: Since, the permutation value of the selected digit '2' has been determined thus remove it from above increasing order. Hence, numeric order of remaining digits is

$$2 \rightarrow 5 *$$

According to given number '5252', third (next) digit is '5'. Hence select digit '5' (as labelled) from above numeric order to find out the values of $Formerity\ (F),\ Similarity\ (S) \&\ Permuty\ (P)$ of selected digit '5' of '5252' & thus permutation value as follows

$$\Rightarrow F_3 = No.of\ dissimilar\ digits\ appearing\ before\ selected\ digit\ '5' of\ 5252\ in\ above\ order$$

$$= 1 \qquad (there\ is\ only\ one\ digit\ '2' dissimilar\ \&\ appearing\ before\ '5')$$

$$\Rightarrow S_3 = No.of\ digits\ similar\ to\ selected\ digit\ '5'\ from\ 5252\ in\ above\ numeric\ order,$$

$$including\ itself$$

$$= 1 \qquad (there\ is\ only\ one\ digit\ '5'\ which\ is\ similar\ to\ itself)$$

$$\Rightarrow P_3 = No.of\ permutations\ obtained\ from\ remaining\ digits\ that\ is\ only\ '2'$$

$$(\ excluding\ selected\ digit\ '5'\ )\ in\ above\ numeric\ order$$

$$= 1! = 1$$

$$\therefore permutation\ value\ of\ selected\ digit\ '5',\ (P_V)_3 = F_3\left(\frac{P_3}{S_3}\right)$$

$$= 1\left(\frac{1}{1}\right) = 1$$

Step 5: Since the permutation value of selected digit '5' has been determined thus remove it from above numeric order. Hence, numeric order of remaining digits is

$$2\ *$$

Since, '2' is the fourth & last digit (as labelled) of the number 5252 hence its permutation value is 1

$$\Rightarrow (P_V)_4 = 1$$

Thus, rank of '5252' in increasing order is given as follows

$$\Rightarrow R(5252 \uparrow) = (P_V)_1 + (P_V)_2 + (P_V)_3 + (P_V)_4$$

$$= 3 + 0 + 1 + 1 = 5$$

Given number '5252' has total no. of digits $n = 4$

No. of the repetitive digits, $p = 2\ (digit\ '2')$& $\quad q = 2\ (digit\ '5')$

Total no. of the numbers formed is given as

$$N_N = \frac{4!}{2!\,2!} = \frac{24}{2 \times 2} = 6$$

The first & last numbers of the series are obtained by arranging all the digits in certain numeric order as follows

$$first\ number \rightarrow 2255 \qquad (arranging\ all\ the\ digits\ in\ increasing\ order)$$

$$last\ number \rightarrow 5522 \qquad (arranging\ all\ the\ digits\ in\ decreasing\ order)$$

Thus, all the numbers can be arranged in increasing/decreasing numeric order as follows

Number	Rank in Increasing Order (↑)	Rank in decreasing Order (↓)
2255	1	6
2525	2	5
2552	3	4
5225	4	3
5252	5	2
5522	6	1

Similarly, rank of '2552' in increasing order can be determine as follows

Arrange all the digits, keeping repetitive digits together, in increasing order as follows

$$2 \rightarrow 2 \rightarrow 5 \rightarrow 5$$

$$\Rightarrow R(5225 \uparrow) = (P_V)_1 + (P_V)_2 + (P_V)_3 + (P_V)_4$$

$$= F_1\left(\frac{P_1}{S_1}\right) + F_2\left(\frac{P_2}{S_2}\right) + F_3\left(\frac{P_3}{S_3}\right) + F_4\left(\frac{P_4}{S_4}\right)$$

$$= 2\left(\frac{\left(\frac{3!}{2!}\right)}{2}\right) + 0\left(\frac{2!}{2}\right) + 0\left(\frac{1!}{1}\right) + 1 = 3 + 0 + 0 + 1 = 4$$

$$\Rightarrow R(5225 \downarrow) = N_N - R(5225 \uparrow) + 1 = \frac{4!}{2!\,2!} - 4 + 1 = 6 - 4 + 1 = 3$$

Similarly, rank of '2552' in increasing order can be determine as follows

Arrange all the digits, keeping repetitive digits together, in increasing order as follows

$$2 \to 2 \to 5 \to 5$$

Select all the digits one by one from above increasing order, according to given (original) number 5225, to find permutation values & then add them to find out the rank of given number

$$\Rightarrow R(5522 \uparrow) = (P_V)_1 + (P_V)_2 + (P_V)_3 + (P_V)_4$$

$$= F_1\left(\frac{P_1}{S_1}\right) + F_2\left(\frac{P_2}{S_2}\right) + F_3\left(\frac{P_3}{S_3}\right) + F_4\left(\frac{P_4}{S_4}\right)$$

$$= 2\left(\frac{\left(\frac{3!}{2!}\right)}{2}\right) + 2\left(\frac{2!}{2}\right) + 0\left(\frac{1!}{1}\right) + 1 = 3 + 2 + 0 + 1 = 6$$

$$\Rightarrow R(5522 \downarrow) = N_N - R(5522 \uparrow) + 1 = \frac{4!}{2!\,2!} - 6 + 1 = 6 - 6 + 1 = 1$$

Note: By following the above procedure, rank of any number having non-zero digits (repetitive or non-repetitive) can be determined.

Example 2: Let's take the following example of non-zero digits 2, 3, 3, 6, 6, 8, 9

Find out the rank of the number '6823369' in increasing and decreasing order

Arrange all the digits, keeping repetitive digits together, in increasing order as follows

$$2 \to 3 \to 3 \to 6 \to 6 \to 8 \to 9$$

Now, select all the digits, according to 6823369, from above numeric order to find out permutation value of each of selected digits. Add permutation values of all the digits to find rank in increasing order as follows

$$\Rightarrow R(6823369 \uparrow) = F_1\left(\frac{P_1}{S_1}\right) + F_2\left(\frac{P_2}{S_2}\right) + F_3\left(\frac{P_3}{S_3}\right) + F_4\left(\frac{P_4}{S_4}\right) + F_5\left(\frac{P_5}{S_5}\right) + F_6\left(\frac{P_6}{S_6}\right) + F_7\left(\frac{P_7}{S_7}\right)$$

$$= 3\left(\frac{\left(\frac{6!}{2!}\right)}{2}\right) + 4\left(\frac{\left(\frac{5!}{2!}\right)}{1}\right) + 0\left(\frac{\left(\frac{4!}{2!}\right)}{1}\right) + 0\left(\frac{3!}{2}\right) + 0\left(\frac{2!}{1}\right) + 0\left(\frac{1!}{1}\right) + 1$$

$$= 540 + 240 + 0 + 0 + 0 + 0 + 1 = 781$$

$$\Rightarrow R(6823369 \downarrow) = N_N - R(6823369 \uparrow) + 1 = \frac{7!}{2!\,2!} - 781 + 1$$

$$= 1260 - 781 + 1 = 480$$

While total no. of the numbers (permutations) formed is given as

$$N_N = \frac{7!}{2!\,2!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1} = 1260$$

The first & last numbers of the series are obtained by arranging all the digits in certain numeric order as follows

$$first\ number \to 2336689 \qquad (arranging\ all\ the\ digits\ in\ increasing\ order)$$

$$last\ number \rightarrow 9866332 \qquad (arranging\ all\ the\ digits\ in\ decreasing\ order)$$

Thus, all the numbers can be arranged in increasing/decreasing numeric orders as follows

The ranks of given number 6823369 has been labelled (*) in the number series below:

Number	Rank in Increasing Order(↑)	Rank in Decreasing Order(↓)
2336689	1	1260
2336698	2	1259
2336869	3	1258
2336896	4	1257
2336968	5	1256
…………… …………… ……………	…………… …………… ……………	…………… …………… ……………
6698332	780	481
6823369*	781*	480*
6823396	782	479
…………… …………… ……………	…………… …………… ……………	…………… …………… ……………
9863623	1256	5
9863632	1257	4
9866233	1258	3
9866323	1259	2
9866332	1260	1

Similarly, ranks of the number 6698332 can be found out as follows

Arrange all the digits, keeping repetitive digits together, in increasing order as follows

$$2 \rightarrow 3 \rightarrow 3 \rightarrow 6 \rightarrow 6 \rightarrow 8 \rightarrow 9$$

Now, select all the digits, according to 6698332, from above numeric order to find out permutation value of each of selected digits. Add permutation values of all the digits to find rank in increasing order as follows

$$\Rightarrow R(6698332 \uparrow) = F_1\left(\frac{P_1}{S_1}\right) + F_2\left(\frac{P_2}{S_2}\right) + F_3\left(\frac{P_3}{S_3}\right) + F_4\left(\frac{P_4}{S_4}\right) + F_5\left(\frac{P_5}{S_5}\right) + F_6\left(\frac{P_6}{S_6}\right) + F_7\left(\frac{P_7}{S_7}\right)$$

$$= 3\left(\frac{\left(\frac{6!}{2!}\right)}{2}\right) + 3\left(\frac{\left(\frac{5!}{2!}\right)}{1}\right) + 4\left(\frac{\left(\frac{4!}{2!}\right)}{1}\right) + 3\left(\frac{\left(\frac{3!}{2!}\right)}{1}\right) + 1\left(\frac{2!}{2}\right) + 1\left(\frac{1!}{1}\right) + 1$$

$$= 540 + 180 + 48 + 9 + 1 + 1 + 1 = 780$$

$$\Rightarrow R(6698332 \downarrow) = N_N - R(6698332 \uparrow) + 1 = \frac{7!}{2!\,2!} - 780 + 1$$

$$= 1260 - 780 + 1 = 481$$

Similarly, rank of number 9866323 can determined as follows

Arrange all the digits, keeping repetitive digits together, in increasing order as follows

$$2 \rightarrow 3 \rightarrow 3 \rightarrow 6 \rightarrow 6 \rightarrow 8 \rightarrow 9$$

Now, select all the digits, according to 6698332, from above numeric order to find out permutation value of each of selected digits. Add permutation values of all the digits to find rank in increasing order as follows:

$$\Rightarrow\ R(9866323\uparrow) = F_1\left(\frac{P_1}{S_1}\right) + F_2\left(\frac{P_2}{S_2}\right) + F_3\left(\frac{P_3}{S_3}\right) + F_4\left(\frac{P_4}{S_4}\right) + F_5\left(\frac{P_5}{S_5}\right) + F_6\left(\frac{P_6}{S_6}\right) + F_7\left(\frac{P_7}{S_7}\right)$$

$$= 6\left(\frac{\left(\frac{6!}{2!2!}\right)}{1}\right) + 5\left(\frac{\left(\frac{5!}{2!2!}\right)}{1}\right) + 3\left(\frac{\left(\frac{4!}{2!}\right)}{2}\right) + 3\left(\frac{\left(\frac{3!}{2!}\right)}{1}\right) + 1\left(\frac{2!}{2}\right) + 0\left(\frac{1!}{1}\right) + 1$$

$$= 1080 + 150 + 18 + 9 + 1 + 0 + 1 = 1259$$

$$\Rightarrow\ R(9866323\downarrow) = N_N -\ R(9866323\uparrow) + 1 = \frac{7!}{2!\,2!} - 1259 + 1$$

$$= 1260 - 1259 + 1 = 2$$

Example 3: Find out increasing and decreasing order (rank) of number 8713273 in the group of all the numbers, arranged in increasing order, obtained by permuting all the digits together.

Sol. Arrange all the digits, keeping repetitive digits together, in increasing order as follows

$$1 \rightarrow 2 \rightarrow 3 \rightarrow 3 \rightarrow 7 \rightarrow 7 \rightarrow 8$$

Now, select all the digits, according to 8713273, from above numeric order to find out permutation value of each of selected digits. Add permutation values of all the digits to find rank in increasing order as follows

$$\Rightarrow\ R(8713273\uparrow) = F_1\left(\frac{P_1}{S_1}\right) + F_2\left(\frac{P_2}{S_2}\right) + F_3\left(\frac{P_3}{S_3}\right) + F_4\left(\frac{P_4}{S_4}\right) + F_5\left(\frac{P_5}{S_5}\right) + F_6\left(\frac{P_6}{S_6}\right) + F_7\left(\frac{P_7}{S_7}\right)$$

$$= 6\left(\frac{\left(\frac{6!}{2!2!}\right)}{1}\right) + 4\left(\frac{\left(\frac{5!}{2!}\right)}{2}\right) + 0\left(\frac{\left(\frac{4!}{2!}\right)}{1}\right) + 1\left(\frac{3!}{2}\right) + 0\left(\frac{2!}{1}\right) + 1\left(\frac{1!}{1}\right) + 1$$

$$= 1080 + 120 + 0 + 3 + 0 + 1 + 1 = 1205$$

Decreasing order of

$$\Rightarrow\ R(8713273\downarrow) = N_N -\ R(8713273\uparrow) + 1 = \frac{7!}{2!\,2!} - 1205 + 1$$

$$= 1260 - 1205 + 1 = 56$$

Thus, increasing order of number '8713273' is 1205 out of total 1260 numbers and decreasing order is 56 in the same group (arrangement).

4. NUMBERS HAVING BOTH ZERO & NON-ZERO DIGITS

In a certain group of digits (both zero & non-zero), let

$$n = no.of\ non\ zero\ digits, out\ of\ which\ no.of\ repetitive\ digits\ are\ p, q, r, s, \dots\dots\dots$$

$$n_o = no.of\ zero\ digits$$

$$\therefore\ total\ no.of\ the\ digits\ (zero + non - zero) = n + n_o$$

In this case, if all the digits are permuted together then we get two types of number

4.1 SIGNIFICANT NUMBER: The number having first digit as non-zero (other than zero)

4.2 NON-SIGNIFICANT NUMBER: The number having first digit as zero

Now, if first digit is selected as zero then no. of the remaining digits is $(n+n_o-1)$ out of which no. of the repetitive digits are $(n_o-1),\ p,\ q,\ r, \ldots\ldots\ldots$ then

The total (N_o) no. of the non-significant numbers is given as

$$\Rightarrow N_o = \frac{(n+n_o-1)!}{(n_o-1)!\,p!\,q!\,r!\ldots\ldots\ldots}$$

While total no. (N) of the significant numbers is given as

$$\Rightarrow \quad N = no. of\ all\ the\ numbers - no. of\ non\ significant\ numbers$$

$$= \frac{(n+n_o)!}{n_o!\,p!\,q!\,r!\ldots\ldots\ldots} - \frac{(n+n_o-1)!}{(n_o-1)!\,p!\,q!\,r!\ldots\ldots\ldots}$$

$$= \left(\frac{n+n_o}{n_o}-1\right)\frac{(n+n_o-1)!}{(n_o-1)!\,p!\,q!\,r!\ldots\ldots\ldots} = \frac{n(n+n_o-1)!}{n_o!\,p!\,q!\,r!\ldots\ldots\ldots}$$

Now, assuming all the digits as non-zero (significant), find the rank of the given number using HCR's Formula by following normal procedure

If we subtract the value N_o (no. of non-significant numbers) from the rank obtained by HCR's Formula applied for all $(n+n_o)$ digits then we find the actual numeric order (increasing or decreasing) of any number having both zero & non-zero digits as follows

$$R(number) = \sum_{i=1}^{i=n+n_o} F_i\left(\frac{P_i}{S_i}\right) - N_o$$

$$= \sum_{i=1}^{i=n+n_o} F_i\left(\frac{P_i}{S_i}\right) - \frac{(n+n_o-1)!}{(n_o-1)!\,p!\,q!\,r!\ldots\ldots\ldots}$$

If there is no zero digits then on setting $n_o = 0$ in the above formula, we get

$$R(number) = \sum_{i=1}^{i=n+0} F_i\left(\frac{P_i}{S_i}\right) - \frac{(n+0-1)!}{(0-1)!\,p!\,q!\,r!\ldots\ldots\ldots}$$

$$= \sum_{i=1}^{i=n} F_i\left(\frac{P_i}{S_i}\right) \pm 0 = \sum_{i=1}^{i=n} F_i\left(\frac{P_i}{S_i}\right)$$

This is equally applicable for all the alphabetic words & numbers having non-zero digits.

4.3 ILLUSTRATIVE EXMAPLES

Example 1: Let there be a group of digits say 2, 0, 0, 0, 3, 3, 4, 7, if all the digits are permuted together & all the significant numbers (8-digit numbers) obtained are arranged in increasing order, find the rank of any number say '40307203'

Sol. Here in the given number '40307203'

$$n = no. of\ non\ zero\ digits = 5$$

$$p = no.of\ repetitive\ digits = 2 \quad (digit\ '3'\ repeats\ 2\ times)$$

$$n_o = no.of\ zero\ digits = 3$$

$$\therefore total\ no.of\ the\ digits\ (zero + non\ zero) = n + n_o = 5 + 3 = 8$$

The total (N_o) no. of the non-significant numbers is given as

$$\Rightarrow N_o = \frac{(n + n_o - 1)!}{(n_o - 1)!\, p!\, q!\, r! \dots \dots \dots}$$

$$= \frac{(5 + 3 - 1)!}{(3 - 1)!\, 2!} = \frac{7!}{2!\, 2!} = 1260$$

While total no. (N) of the significant numbers is given as

$$\Rightarrow N = \frac{n(n + n_o - 1)!}{n_o!\, p!\, q!\, r! \dots \dots \dots} = \frac{5(5 + 3 - 1)!}{3!\, 2!}$$

$$= \frac{5 \times 7!}{3!\, 2!} = 2100$$

Now, arrange all the digits in the increasing order, keeping repetitive digits together as follows

$$0 \to 0 \to 0 \to 2 \to 3 \to 3 \to 4 \to 7$$

Now, assuming all zero digits as non-zero & applying HCR's Rank formula i.e. finding the permutation values of all the digits by following same procedure & adding them together as follows

$$\sum_{i=1}^{i=8} F_i \left(\frac{P_i}{S_i}\right)$$

$$= F_1\left(\frac{P_1}{S_1}\right) + F_2\left(\frac{P_2}{S_2}\right) + F_3\left(\frac{P_3}{S_3}\right) + F_4\left(\frac{P_4}{S_4}\right) + F_5\left(\frac{P_5}{S_5}\right) + F_6\left(\frac{P_6}{S_6}\right) + F_7\left(\frac{P_7}{S_7}\right) + F_8\left(\frac{P_8}{S_8}\right)$$

$$= 6\left(\frac{\left(\frac{7!}{3!2!}\right)}{1}\right) + 0\left(\frac{\left(\frac{6!}{2!2!}\right)}{3}\right) + 3\left(\frac{\left(\frac{5!}{2!}\right)}{2}\right) + 0\left(\frac{4!}{2}\right) + 3\left(\frac{3!}{1}\right) + 1\left(\frac{2!}{1}\right) + 0\left(\frac{1!}{1}\right) + 1$$

$$= 2520 + 0 + 90 + 0 + 18 + 2 + 0 + 1 = 2631$$

Hence, on setting the values, rank of '40307203' in increasing order

$$\Rightarrow R(40307203 \uparrow) = \sum_{i=1}^{i=8} F_i\left(\frac{P_i}{S_i}\right) - N_o$$

$$= 2631 - 1260 = 1371$$

Hence, rank of number '40307203' in increasing order is 1371 out of 2100 significant numbers.

While all the numbers are arranged in their actual numeric order (increasing or decreasing) which can be verified using HCR's Formula as shown in the table below:

Number	Rank in Increasing Order(↑)	Rank in Decreasing Order(↓)
20003347	1	2100
20003374	2	2099

20003437	3	2098
20003473	4	2097
20003734	5	2096
…………… …………… ……………	……………. ……………. …………….	……………… ……………… ………………
40307032	1370	731
40307203*	1371*	730*
40307230	1372	729
…………… …………… ……………	……………. ……………. …………….	…………….. …………….. ……………..
74323000	2096	5
74330002	2097	4
74330020	2098	3
74330200	2099	2
74332000	2100	1

Example 2: Let there be a group of digits say 1, 0, 0, 0, 0, 2, 2, 3, if all the digits are permuted together & all the significant numbers (8-digit numbers) obtained are arranged in increasing order, find the rank of any number say '20030102'

Sol. Here in the given number '20030102'

$$n = no. of\ non\ zero\ digits = 4$$

$$p = no. of\ repetitive\ digits = 2 \quad (digit\ '2'\ repeats\ 2\ times)$$

$$n_o = no. of\ zero\ digits = 4$$

$$\therefore total\ no. of\ the\ digits\ (zero + non\ zero) = n + n_o = 4 + 4 = 8$$

The total (N_o) no. of the non-significant numbers is given as

$$\Rightarrow N_o = \frac{(n + n_o - 1)!}{(n_o - 1)!\,p!\,q!\,r!\ldots\ldots\ldots.}$$

$$= \frac{(4 + 4 - 1)!}{(4 - 1)!\,2!} = \frac{7!}{3!\,2!} = 420$$

While total no. (N) of the significant numbers (8-digit numbers) is given as

$$\Rightarrow N = \frac{n(n + n_o - 1)!}{n_o!\,p!\,q!\,r!\ldots\ldots\ldots.} = \frac{4(4 + 4 - 1)!}{4!\,2!}$$

$$= \frac{4 \times 7!}{4!\,2!} = 420$$

Now, arrange all the digits in the increasing order, keeping repetitive digits together as follows

$$0 \to 0 \to 0 \to 0 \to 1 \to 2 \to 2 \to 3$$

Now, assuming all zero digits as non-zero & applying HCR's Rank formula i.e. finding the permutation values of all the digits by following same procedure & adding them together as follows

$$\sum_{i=1}^{i=8} F_i\left(\frac{P_i}{S_i}\right)$$

$$= F_1\left(\frac{P_1}{S_1}\right) + F_2\left(\frac{P_2}{S_2}\right) + F_3\left(\frac{P_3}{S_3}\right) + F_4\left(\frac{P_4}{S_4}\right) + F_5\left(\frac{P_5}{S_5}\right) + F_6\left(\frac{P_6}{S_6}\right) + F_7\left(\frac{P_7}{S_7}\right) + F_8\left(\frac{P_8}{S_8}\right)$$

$$= 5\left(\frac{\left(\frac{7!}{4!}\right)}{2}\right) + 0\left(\frac{\left(\frac{6!}{3!}\right)}{4}\right) + 0\left(\frac{\left(\frac{5!}{2!}\right)}{3}\right) + 4\left(\frac{\left(\frac{4!}{2!}\right)}{1}\right) + 0\left(\frac{3!}{2}\right) + 1\left(\frac{2!}{1}\right) + 0\left(\frac{1!}{1}\right) + 1$$

$$= 525 + 0 + 0 + 48 + 0 + 2 + 0 + 1 = 576$$

Hence, on setting the values, rank of '20030102' in increasing order

$$\Rightarrow R(20030102 \uparrow) = \sum_{i=1}^{i=8} F_i\left(\frac{P_i}{S_i}\right) - N_o$$

$$= 576 - 420 = 156$$

Hence, rank of number '40307203' in increasing order is 156 out of 420 significant numbers.

While all the numbers are arranged in their actual numeric order (increasing or decreasing) Which can be verified using HCR's Formula as in the table below,

Number	Rank in Increasing Order(↑)	Rank in Decreasing Order(↓)
10000223	1	420
10000232	2	419
10000322	3	418
10002023	4	417
10002032	5	416
…………… …………… ……………	……………. ……………. …………….	……………… ……………… ………………
20030021	155	266
20030102*	156*	265*
20030120	157	264
…………… …………… ……………	……………. ……………. …………….	…………….. …………….. ……………..
32200001	416	5
32200010	417	4
32200100	418	3
32201000	419	2
32210000	420	1

Example 3: Let there be a group of digits say 0, 0, 0, 4, 7, 9, 9, if all the digits are permuted together & all the significant numbers (7-digit numbers) obtained are arranged in increasing order, find the rank of any number say '9097400'

Sol. Here in the given number '9097400'

$$n = no.\,of\ non\ zero\ digits = 4$$

$$p = no.\,of\ repetitive\ digits = 2 \quad (digit\ '9'\ repeats\ 2\ times)$$

$$n_o = no.of\ zero\ digits = 3$$

$$\therefore total\ no.of\ the\ digits\ (zero + non\ zero) = n + n_o = 4 + 3 = 7$$

The total (N_o) no. of the non-significant numbers is given as

$$\Rightarrow N_o = \frac{(n + n_o - 1)!}{(n_o - 1)!\,p!\,q!\,r!\ldots\ldots\ldots}$$

$$= \frac{(4 + 3 - 1)!}{(3 - 1)!\,2!} = \frac{6!}{2!\,2!} = 180$$

While total no. (N) of the significant numbers (7-digit numbers) is given as

$$\Rightarrow N = \frac{n(n + n_o - 1)!}{n_o!\,p!\,q!\,r!\ldots\ldots\ldots} = \frac{4(4 + 3 - 1)!}{3!\,2!}$$

$$= \frac{4 \times 6!}{3!\,2!} = 240$$

Now, arrange all the digits in the increasing order, keeping repetitive digits together as follows

$$0 \to 0 \to 0 \to 4 \to 7 \to 9 \to 9$$

Now, assuming all zero digits as non-zero & applying HCR's Rank formula i.e. finding the permutation values of all the digits by following same procedure & adding them together as follows

$$\sum_{i=1}^{i=7} F_i\left(\frac{P_i}{S_i}\right)$$

$$= F_1\left(\frac{P_1}{S_1}\right) + F_2\left(\frac{P_2}{S_2}\right) + F_3\left(\frac{P_3}{S_3}\right) + F_4\left(\frac{P_4}{S_4}\right) + F_5\left(\frac{P_5}{S_5}\right) + F_6\left(\frac{P_6}{S_6}\right) + F_7\left(\frac{P_7}{S_7}\right)$$

$$= 5\left(\frac{\left(\frac{6!}{3!}\right)}{2}\right) + 0\left(\frac{\left(\frac{5!}{2!}\right)}{3}\right) + 4\left(\frac{\left(\frac{4!}{2!}\right)}{1}\right) + 3\left(\frac{\left(\frac{3!}{2!}\right)}{1}\right) + 2\left(\frac{\left(\frac{2!}{2!}\right)}{1}\right) + 0\left(\frac{1!}{2}\right) + 1$$

$$= 300 + 0 + 48 + 9 + 2 + 0 + 1 = 360$$

Hence, on setting the values, rank of '9097400' in increasing order

$$\Rightarrow R(9097400 \uparrow) = \sum_{i=1}^{i=7} F_i\left(\frac{P_i}{S_i}\right) - N_o$$

$$= 360 - 180 = 180$$

Hence, rank of number '9097400' in increasing order is 180 out of 240 significant numbers.

While all the numbers are arranged in their actual numeric order (increasing or decreasing) which can be verified using HCR's Formula as shown in the table below

Number	Rank in Increasing Order(↑)	Rank in Decreasing Order(↓)
4000799	1	240
4000979	2	239
4000997	3	238

4007099	4	237
4007909	5	236
…………… …………… ……………	……………… ……………… ………………	…………… …………… ……………
4070099	13	228
4070909	14	227
4070990	15	226
…………… …………… ……………	……………. ……………. …………….	……………… ……………… ………………
9097040	179	62
9097400*	180*	61*
9400079	181	60
…………… …………… ……………	……………. ……………. …………….	…………….. …………….. ……………..
994070	236	5
994700	237	4
997004	238	3
997040	239	2
997400	240	1

5. Linear permutations of non-algebraic articles (things) having different shape, size, colour etc.

Problems of linear permutations are easily simplified by using alphabetic letters or digits but alphabetic letters are more suitable for significant & better distinction among the articles which have at least one easily distinguishable property like their appearance such as shape, size, colour, surface-design etc. & are equally important at all the places in linear arrangements.

Since, we are very usual with the linear sequence of alphabetic-letters hence all the given articles one by one are replaced by alphabetic letters, A, B, C, D, E... ….. according to the sequence. Similar articles are replaced by the same letters while different articles are replaced by the different letters. Then all the linear permutations of different articles can be dealt with ease to find out the rank of any randomly selected linear permutation or to arrange them in correct order while the given articles have a pre-defined linear sequence.

5.1 WORKING STEPS

Under all the conditions of applications of HCR’s Formula

For a given linear permutation (of certain articles equally significant at all the places (positions)) randomly selected from a series of permutations or

For a given group of certain articles equally significant at all the places in the arrangements & any randomly selected linear permutation of all these articles,

Step 1: Arrange all the articles, keeping similar ones (if any) together in a consecutive manner, in a linear sequence/fashion according to the **pre-defined basis of priority** (any easily **distinguishable-property** of the articles among all like their **shape**, **size**, **colour**, **surface-design** etc.). It is called pre-defined sequence of articles equally likely significant at all the places in the arrangements.

Step 2: Now, replace all the articles one by one by an alphabetic letter, A, B, C, D, ……..accordingly, while similar articles are replaced by the same alphabetic letters in the correct sequence. Thus each of the pre-defined linear sequence of the articles is replaced by an equivalent alphabetic linear sequence & hence any random linear permutation is replaced by a certain alphabetic linear permutation. Hence by following the same rule of alphabets, apply “**HCR’s Rank or Series**

Formula" on any of the linear permutation to find its rank (position) in the correct order or to position any linear permutation at the exact position.

5.2Illustrative Examples on linear permutations of non-algebraic articles

5.2.1 Articles having different shapes & sizes

Consider the following articles

$$© ∟ ® \beta\alpha\beta © ©$$

We know that there are total 8 articles out of which three ©& two $\beta$are similar articles. Now, if all these articles are permuted together then the total number of the possible ways of linear arrangements or random permutations (having no sequence) consisting of all the given articles is given as

$$= \frac{8!}{3!\,2!} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 2 \times 1} = 3360$$

We also know that 3360 is the number of all the **random possible linear permutations** formed by permuting all the given articles together.

Now, in order to arrange all these random linear permutations which do not have any correct order of arrangement, hence it is must that we have to predefine a linear sequence of all the articles according to a pre-defined basis of priority. It is obvious that the above articles have different appearance i.e. each of the articles above differs from other ones in shape & size. Thus shape or size (usually both are distinguishable) of all the articles is the most suitable **distinguishable property (basis of priority)**. According to the basis of priority & utility (like usefulness, significance etc.), let the pre-defined linear sequence, keeping similar ones together in consecutive manner, be as follows

$$∟ \rightarrow ® \rightarrow © \rightarrow © \rightarrow © \rightarrow \beta \rightarrow \beta \rightarrow \alpha$$

& now, we select any random linear permutation of the given articles as follows

$$\beta © \alpha © © \beta ® ∟$$

It is difficult to find the rank (correct position) of the above random linear permutation. All the linear permutations of such articles can be dealt with ease by using alphabetic letters & their linear permutations to find out the rank of the above (or any) randomly selected linear permutation. Hence let

$$∟ \equiv A, \quad ® \equiv B, \quad © \equiv C, \quad \beta \equiv D \quad \& \alpha \equiv E$$

Thus, actual sequence of the given objects can be replaced by alphabetic order as follows (this is done only to simplify the problem)

$$A \rightarrow B \rightarrow C \rightarrow C \rightarrow C \rightarrow D \rightarrow D \rightarrow E$$

Now, the random selected permutation of given articles can be replaced as follows

$$[\beta © \alpha © © \beta ® ∟] \equiv [DCECCDBA]$$

In this case, we have

$$Rank\ of\ [\beta © \alpha © © \beta ® ∟] = Rank\ of\ [DCECCDBA]$$

Thus, we are to find the rank of linear permutation "**DCECCDBA**" in the alphabetic order,

Now, the same procedure can be followed as first arrange all the letters in actual alphabetic order as follows

$$A \rightarrow B \rightarrow C \rightarrow C \rightarrow C \rightarrow D \rightarrow D \rightarrow E$$

Use the above **pre-defined linear sequence** to find out the permutation values of each of the letters.

Now, select & remove the letters one by one from alphabetic order according to **DCECCDBA** to find rank as follows (applying "**HCR's Rank or Series Formula**")

$$R(DCECCDBA) = 5\left(\frac{\left(\frac{7!}{3!}\right)}{2}\right) + 2\left(\frac{\left(\frac{6!}{2!}\right)}{3}\right) + 5\left(\frac{\left(\frac{5!}{2!}\right)}{1}\right) + 2\left(\frac{4!}{2}\right) + 2\left(\frac{3!}{1}\right) + 2\left(\frac{2!}{1}\right)$$

$$+1\left(\frac{1!}{1}\right)+1$$

$$= 2100 + 240 + 300 + 24 + 12 + 4 + 1 + 1 = 2682$$

$$\therefore\ Rank\ of\ randomly\ selected\ permutaion[\beta©\alpha©©\beta®∟]$$

$$= Rank\ of\ [DCECCDBA] = 2682$$

All other permutations can be arranged by using alphabetic order as follows

Equivalent Alphabetic Word	Linear Permutation of articles	Rank (Order)
ABCCCDDE	∟®© ©©$\beta\beta\alpha$	1
ABCCCDED	∟®© ©©$\beta\beta\alpha$	2
ABCCCEDD	∟®© ©©$\beta\beta\alpha$	3
ABCCDCDE	∟®© ©©$\beta\beta\alpha$	4
ABCCDCED	∟®© ©©$\beta\beta\alpha$	5
…………… …………… ……………	…………….. …………….. ……………..	……………… ……………… ………………
DCECCDAB	β©α©©β∟®	2681
DCECCDBA*	β©α©©β®∟	2682*
DCECDABC	β©α©β∟®©	2683
…………… …………… ……………	…………….. …………….. ……………..	…………….. …………….. ……………..
EDDCCACB	$\alpha\beta\beta$© ©∟©®	3356
EDDCCBAC	$\alpha\beta\beta$© ©®∟©	3357
EDDCCBCA	$\alpha\beta\beta$© ©®©∟	3358
EDDCCCAB	$\alpha\beta\beta$© ©©∟®	3359
EDDCCCBA	$\alpha\beta\beta$© ©©®∟	3360

5.2.2 Articles having different colours (but identical in shape & size)

Consider the following articles

©©©©©©©©©

We know that there are total 9 articles (identical in shape & size) out of which two are green, three are sky-blue & two are purple articles. Now, if all these articles are permuted together then the total number of the possible ways of linear arrangements or random permutations (having no sequence) consisting of all the given articles is given as

$$= \frac{9!}{3!\,2!\,2!} = \frac{8\times7\times6\times5\times4\times3\times2\times1}{3\times2\times1\times2\times1} = 15120$$

We also know that 15120 is the number of all the **random possible linear permutations** formed by permuting all the given articles together.

Now, in order to arrange all these random linear permutations which do not have any correct order of arrangement, it is must that we have to predefine a linear sequence of all the articles according to a pre-defined basis of priority. It is obvious that the above articles are identical in shape & size, but each of the articles above differs from other ones in colours. Thus "**colour**" among all the articles is the most suitable **distinguishable property (basis of priority)**. According to the basis of priority & utility (like usefulness, significance etc.), let the linear sequence, keeping similar ones together in consecutive manner, be as follows

© → © → © → © → © → © → © → © → ©

& now, we select any random linear permutation of the given articles as follows

©©©©©©©©©

It is difficult to find the rank (correct position) of the above random linear permutation. All the linear permutations of such articles can be dealt with ease by using alphabetic letters & their linear permutations to find out the rank of the above (or any) randomly selected linear permutation. Hence let

$$A \equiv ©, \quad B \equiv ©, \quad C \equiv ©, \quad D \equiv ©\& \quad E \equiv ©$$

Thus, actual sequence of the given objects can be replaced by alphabetic order as follows (this is done only to simplify the problem)

$$A \to B \to B \to C \to C \to C \to D \to D \to E$$

Now, the random selected permutation of given articles can be replaced as follows

$$[©©©©©©©©©] \equiv [DCBAEBCDC]$$

In this case, we have

$$Rank\ of\ [©©©©©©©©©] = Rank\ of\ [DCBAEBCDC]$$

Thus, we are to find the rank of linear permutation "**DCBAEBCDC**" in the alphabetic order,

Now, the same procedure can be followed as first arrange all the letters in actual alphabetic order as follows

$$A \to B \to B \to C \to C \to C \to D \to D \to E$$

Use the above **pre-defined linear sequence** to find out the permutation values of each of the letters.

Now, select & remove the letters one by one from alphabetic order according to **DCBAEBCDC** to find rank as follows (applying "**HCR's Rank or Series Formula**")

$$R(DCBAEBCDC) = 6\left(\frac{\left(\frac{8!}{3!2!}\right)}{2}\right) + 3\left(\frac{\left(\frac{7!}{2!2!}\right)}{3}\right) + 1\left(\frac{\left(\frac{6!}{2!}\right)}{2}\right) + 0\left(\frac{\left(\frac{5!}{2!}\right)}{1}\right) + 4\left(\frac{\left(\frac{4!}{2!}\right)}{1}\right)$$

$$+0\left(\frac{\left(\frac{3!}{2!}\right)}{1}\right) + 0\left(\frac{2!}{2}\right) + 1\left(\frac{1!}{1}\right) + 1$$

$$= 10080 + 1260 + 180 + 0 + 48 + 0 + 0 + 1 + 1 = 11570$$

$$\therefore\ Rank\ of\ randomly\ selected\ permutaion[©©©©©©©©©]$$

$$= Rank\ of\ [DCBAEBCDC] = 11570$$

All other permutations can be arranged by using alphabetic order as follows:

Equivalent Alphabetic Word	Linear Permutation of articles	Rank (Order)
ABBCCCDDE	©©©©©©©©©	1
ABBCCCDED	©©©©©©©©©	2
ABBCCCEDD	©©©©©©©©©	3
ABBCCDCDE	©©©©©©©©©	4
ABBCCDCED	©©©©©©©©©	5
…………… …………… ……………	……………. ……………. …………….	……………… ……………… ………………
DCBAEBCCD	©©©©©©©©©	11569
DCBAEBCDC*	©©©©©©©©©	11570*
DCBAEBDCC	©©©©©©©©©	11571
…………… …………… ……………	……………. ……………. …………….	…………….. …………….. ……………..
EDDCCBCAB	©©©©©©©©©	15116

EDDCCBCBA	©©©©©©©©©	15117
EDDCCCABB	©©©©©©©©©	15118
EDDCCCBAB	©©©©©©©©©	15119
EDDCCCBBA	©©©©©©©©©	15120

5.2.3 Articles which are similar & dissimilar in shape, size, colour, surface-design etc.

Consider the following articles

®¥ε¥€§¥©βε∃:©

We know that there are total 13 articles, out of which two are green& two are black articles which are similar to each other. Articles which are similar in shape and size but different in colour are considered as different articles. Now, if all these articles are permuted together then the total number of the possible ways of linear arrangements or random permutations (having no sequence) consisting of all the given articles is given as

$$= \frac{13!}{2!\,2!} = \frac{13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1} = 1556755200$$

We also know that 1556755200 is the number of all the **random possible linear permutations** formed by permuting all the given articles together.

Now, in order to arrange all these random linear permutations which do not have any correct order of arrangement, it is must that we have to pre-define a linear sequence of all the articles according to a pre-defined basis of priority. It is obvious that the above articles are similar & dissimilar in shape & size, colour etc. Here, it is difficult to identify the most suitable **distinguishable property (basis of priority)** among all these non-homogeneous articles (of different categories).But, all these articles can be easily distinguished by their relative appearances. Now, according to the basis of utility (like usefulness, significance etc.), let the linear sequence, keeping similar ones together in consecutive manner, be as follows

® → ¥ → ¥ → ¥ → € → § → β → ε → ε → ∃ → : → © → ©

& now, we select any random linear permutation of the given articles as follows

:©¥∃¥€β¥§ε©®ε

It is difficult to find the rank (correct position) of the above random linear permutation. All the linear permutations of such articles can be dealt with ease by using alphabetic letters & their linear permutations to find out the rank of the above (or any) randomly selected linear permutation. Hence let

$A \equiv$ ®, $B \equiv$ ¥, $C \equiv$ ¥, $D \equiv$ €, $E \equiv$ §, $F \equiv \beta$, $G \equiv \varepsilon$, $H \equiv \varepsilon$, $I \equiv \exists$

$J \equiv$:, $K \equiv$ ©

Thus, actual sequence of the given objects can be replaced by alphabetic order as follows (this is done only to simplify the problem)

$$A \to B \to B \to C \to D \to E \to F \to G \to H \to I \to J \to K \to K$$

Now, the random selected permutation of given articles can be replaced as follows

[:©¥∃¥€β¥§ε©®ε] ≡ $[JKBICDFBEGKAH]$

In this case, we have

$Rank\ of$ [:©¥∃¥€β¥§ε©®ε] = $Rank\ of\ [JKBICDFBEGKAH]$

Thus, we are to find the rank of linear permutation "$JKBICDFBEGKAH$" in the alphabetic order,

Now, the same procedure of alphabetic words can be followed as first arrange all the letters in actual alphabetic order as follows

$$A \to B \to B \to C \to D \to E \to F \to G \to H \to I \to J \to K \to K$$

Use the above **pre-defined linear sequence** to find out the permutation values of each of the letters.

Now, select & remove the letters one by one from alphabetic order according to **JKBICDFBEGKAH** to find rank asfollows (applying "**HCR's Rank or Series Formula**")

$$R(JKBICDFBEGKAH) = 10\left(\frac{\left(\frac{12!}{2!2!}\right)}{1}\right) + 10\left(\frac{\left(\frac{11!}{2!}\right)}{2}\right) + 1\left(\frac{10!}{2}\right) + 8\left(\frac{9!}{1}\right) + 2\left(\frac{8!}{1}\right) + 2\left(\frac{7!}{1}\right) + 3\left(\frac{6!}{1}\right)$$

$$+ 1\left(\frac{5!}{1}\right) + 1\left(\frac{4!}{1}\right) + 1\left(\frac{3!}{1}\right) + 2\left(\frac{2!}{1}\right) + 0\left(\frac{1!}{1}\right) + 1$$

$$= 1197504000 + 99792000 + 1814400 + 2903040 + 80640 + 10080 + 2160 + 120 + 24 + 6 + 4 + 0 + 1$$
$$= 1302106475$$

$$\therefore\ Rank\ of\ randomly\ selected\ permutaion[\text{:©¥∃¥€}\beta\text{¥§}\varepsilon\text{©®}\varepsilon]$$

$$= Rank\ of\ [JKBICDFBEGKAH] = 1302106475$$

All other permutations can be arranged by estimating the ranks (alphabetic orders) as tabulated below

EquivalentAlphabetic Word	Linear Permutation of articles	Rank (Order)
ABBCDEFGHIJKK	®¥¥¥€§βεε∃:©©	1
ABBCDEFGHIKJK	®¥¥¥€§βεε∃© :©	2
ABBCDEFGHIKKJ	®¥¥¥€§βεε∃©© :	3
ABBCDEFGHJIKK	®¥¥¥€§βεε:∃©©	4
ABBCDEFGHJKIK	®¥¥¥€§βεε:© ∃©	5
…………… …………… ……………	…………… …………… ……………	…………… …………… ……………
JKBICDFBEGHKA	:©¥∃¥€β¥§εε ©®	1302106474
JKBICDFBEGKAH*	:©¥∃¥€β¥§ε©®ε	1302106475*
JKBICDFBEGKHA	:©¥∃¥€β¥§ε©ε®	1302106476
…………… …………… ……………	…………… …………… ……………	…………… …………… ……………
KKJIHGFEDBCAB	©© :∃εεβ§€¥¥®¥	1556755196
KKJIHGFEDBCBA	©© :∃εεβ§€¥¥¥®	1556755197
KKJIHGFEDCABB	©© :∃εεβ§€¥®¥¥	1556755198
KKJIHGFEDCBAB	©© :∃εεβ§€¥¥®¥	1556755199
KKJIHGFEDCBBA	©© :∃εεβ§€¥¥¥®	1556755200

Thus, we find that by the use of alphabetic words, we can solve the problem of randomly selected linear permutations of certain things. It is sufficient to study the linear permutations of alphabetic letters & digits. We apply the same procedure of words & numbers on the all the linear permutations of certain articles to find their respective rank (correct position). Now, let's generalise HCR's formula for alphabetic words & numbers as follows

6. GENERALISED FORM OF "HCR'S RANK OR SERIES FORMULA"

for alphabetic words & the numbers

$$R(word\ or\ number) = \sum_{i=1}^{i=n+n_o} F_i\left(\frac{P_i}{S_i}\right) - \left[\frac{(n+n_o-1)!}{(n_o-1)!\,p!\,q!\,r!\dots\dots\dots}\right]$$

Where,

$$n = no.of\ significant\ digits/letters$$

$$Out\ of\ which\ no.of\ repetitive\ digits/letters\ are\ 'p','q','r'\ \dots\dots\dots$$

$$n_o = no.of\ non\ significant\ digits/letters\ which\ don't\ appear$$

$$at\ first\ place\ (left\ most)\ of\ a\ number\ or\ a\ word$$

Deduction 01: Rank of Alphabetic Word & Positive Integral Number with non-zero digits

If a given word/positive integral number has $'n'$ no. of the letters, out of which no. of the repetitive letters/non-zero digits are $p, q, r, s, \dots$.. then

In this case, $n_o = 0$

Rank of a word/number is given as

$$R(word\ or\ number) = \sum_{i=1}^{i=n} F_i \left(\frac{P_i}{S_i}\right)$$

Note: If all theletters or non-zero digits in a given word or positive integral number are non-repetitive, then the similarity of all the letters or digits is equal to unity. Hence we have

$S_1 = S_2 = S_3 = \dots\dots\dots\dots\dots\dots\dots. = S_{n-2} = S_{n-1} = S_n = 1 \Rightarrow S_i = 1$Hence, the rank of such word or number is given as follows

$$R(word\ or\ number) = \sum_{i=1}^{i=n} F_i \left(\frac{P_i}{1}\right) = \left|\sum_{i=1}^{i=n} F_i P_i\right|_{non\ repeatability}$$

Deduction 02: Rank of Positive Integral Number with zero & non-zero digits

If a positive integral number has 'n' no. of non-zero digits, out of which no. of repetitive digits are $'p', 'q', 'r' \dots\dots$...&$'n_o'$ no. of the zero digits then

Rank of the number is given as

$$R(number) = \sum_{i=1}^{i=n+n_o} F_i \left(\frac{P_i}{S_i}\right) - \left[\frac{(n+n_o-1)!}{(n_o-1)!\,p!\,q!\,r!\dots\dots\dots.}\right]$$

7. HCR'S AXIOM

"If a given word/positive integral number has total $'n'$ number of the letters/non-zero digits, out of which numbers of repetitive letters/non-zero digits are $p, q, r, s, \dots\dots\dots$.. then total number (N) of the words/numbers formed by permuting all the letters/non-zero digits together is always equal to the rank of last word/number in the actual alphabetic/numeric order"

Mathematically, it is expressed as follows

$$N = \frac{n!}{p!\,q!\,r!\,s!\dots\dots} = \left|\sum_{i=1}^{i=n} F_i \left(\frac{P_i}{S_i}\right)\right|_{last\ word\ /number}$$

It is well known that when the words/numbers, obtained by permuting the latters/non-zero digits together, are arranged in their alphabetic/numeric order then the order of last word/number will always be equal to the total no. of words/numbers (permutations).

Note: Above axiom is based on HCR's Rank Formula used to verify that all the results obtained by the formula are correct.

7.1 Illustrative Examples on HCR's Axiom (Verification of HCR's Formula)

Example 1: Find out total no. of words obtained by permuting the letters A, E, C, F, C, H,together.

Sol. Arrange all the letters in alphabetic order as follows

$$A \to C \to C \to E \to F \to H$$

By arranging all the letters in reverse alphabetic order as follows

Last word: HFECCA

Now, using **HCR's Rank or Series Formula** as follows

$$R(HFECCA)$$

$$= F_1\left(\frac{P_1}{S_1}\right) + F_2\left(\frac{P_2}{S_2}\right) + F_3\left(\frac{P_3}{S_3}\right) + F_4\left(\frac{P_4}{S_4}\right) + F_5\left(\frac{P_5}{S_5}\right) + F_6\left(\frac{P_6}{S_6}\right) + F_7\left(\frac{P_7}{S_7}\right)$$

$$= 5\left(\frac{\left(\frac{5!}{2!}\right)}{1}\right) + 4\left(\frac{\left(\frac{4!}{2!}\right)}{1}\right) + 3\left(\frac{\left(\frac{3!}{2!}\right)}{1}\right) + 1\left(\frac{2!}{2}\right) + 1\left(\frac{1!}{1}\right) + 1$$

$$= 300 + 48 + 9 + 1 + 1 + 1 = 360$$

While total no. of the words in the group (Word Series) is given as follows

$$N_N = \frac{6!}{2!} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = 360$$

Thus, we find that both the results obtained are equal hence **HCR's Axiom** is truewhich verifies the formula.

Example 2: Find out total no. of positive integral numbers obtained by permuting the non-zero digits 6, 6, 4, 4, 1, 4, 8, 9, 7,together.

Sol. Arrange all the digits in increasing numeric order as follows

$$1 \to 4 \to 4 \to 4 \to 6 \to 6 \to 7 \to 8 \to 9$$

By arranging all the digits in decreasing order as follows

Last number: 987664441 (while all the numbers are arranged in increasing order)

Now, using **HCR's Rank or Series Formula** as follows

$$R(987664441 \uparrow)$$

$$= F_1\left(\frac{P_1}{S_1}\right) + F_2\left(\frac{P_2}{S_2}\right) + F_3\left(\frac{P_3}{S_3}\right) + F_4\left(\frac{P_4}{S_4}\right) + F_5\left(\frac{P_5}{S_5}\right) + F_6\left(\frac{P_6}{S_6}\right) + F_7\left(\frac{P_7}{S_7}\right) + F_8\left(\frac{P_8}{S_8}\right) + F_9\left(\frac{P_9}{S_9}\right)$$

$$= 8\left(\frac{\left(\frac{8!}{3!2!}\right)}{1}\right) + 7\left(\frac{\left(\frac{7!}{3!2!}\right)}{1}\right) + 6\left(\frac{\left(\frac{6!}{3!2!}\right)}{1}\right) + 4\left(\frac{\left(\frac{5!}{3!}\right)}{2}\right) + 4\left(\frac{\left(\frac{4!}{3!}\right)}{1}\right) + 1\left(\frac{\left(\frac{3!}{2!}\right)}{3}\right) + 1\left(\frac{2!}{2}\right) + 1\left(\frac{1!}{1}\right) + 1$$

$$= 26880 + 2940 + 360 + 40 + 16 + 1 + 1 + 1 + 1 = 30240$$

While total no. of positive integral numbers in the group (Number Series) is given as follows

$$N_N = \frac{9!}{3!\,2!} = \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 2 \times 1} = 30240$$

Thus, we find that both the results obtained are equal hence **HCR's Axiom** is true which verifies the formula.

8. CONDITIONS OF APPLICATION OF FORMULA

It can be applied to find out the rank (position) of any randomly selected permutation, consisting of certain articles having a **pre-defined linear sequence**, if and only if

1. All the articles have at least one easily distinguishable property among them & are equally significant at all the places (positions) in the arrangements
2. All the articles are permuted together without any condition like **mutual-combinations** (such as **pairs)** of articles, **significance**&**non-significance** of few or certain articles &
3. All the permutations obtained are assumed to be equally **significant**& arranged in a mathematically correct order/sequence.

9. APPLICATIONS

HCR's Formula is equally applicable to find out

a) Alphabetic order of any word **randomly selected** from a correct **alphabetic arrangement** of all the words obtained by permutation which have equal and identical letters
b) Numeric (increasing or decreasing) order of any number **randomly selected** from a correct **numeric arrangement** of all obtained by permutation which have equal and identical digits.
c) Total no. of the words lying between any two words **randomly selected** from a correct alphabetic arrangement of all the words obtained by permutation which have equal and identical letters
d) Total numbers lying between any two numbers **randomly selected** from a correct numeric (increasing or decreasing) arrangement of all the numbers obtained by permutation which have equal and identical digits.
e) Order (position) of any non-algebraic **linear permutation** (consisting of certain **similar** and **dissimilar articles (things)**) randomly selected from a group of all the permutations correctly arranged according to the **pre-defined linear sequence.**
f) **HCR's Rank Formula** is the **most useful formula** for exactly or correctly arranging a few or all the permutations consisting of smaller, larger or very larger no. of equal & identical articles which is practically not possible by any other method in mathematics i.e. **HCR's Formula** is the **mathematically correct** formula to arrange few or all the permutations (of algebraic or non-algebraic or both the articles) in a correct order according to the **pre-defined linear sequence**.

10. CONCLUSION

This formula is equally applicable for all the linear permutations like alphabetic words, numbers & all other linear permutations of certain articles. It is used for finding the rank (correct order of priority) of any randomly selected (or a given) linear permutation (consisting of certain articles having at least one easily distinguishable property like shape, size, colour, surface-design etc.) from the group possessing all possible linear arrangements which are obtained by permuting all the articles together.The same procedure of alphabetic words is followed in order to deal with the problems of all the linear permutations of non-algebraic articles since the use of alphabetic letters simplifies the complex linear permutations.

REFERENCES

It is an original research work in Algebra dealing with the linear permutations like words and positive integral numbers etc.It has no part from any other source. Although, this paper is subjected to the peer review by

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It is an outcome of studied, knowledge, experience & prolonged research work with experiments/tests carried out by the author Mr Harish Chandra Rajpoot. He is a published author of 'Advanced Geometry' with Notion Press, Chennai-600005 India. His book is based on research articles in Applied Mathematics& Physics.